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Coordinated Control Systems Design  
for Fuel Efficiency

Bolt Beranek and Newman Inc.  
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performance indices do not conflict with each other. In particular, we have defined the notion of noninteraction for quadratic cost functionals and found necessary and sufficient conditions for noninteraction between quadratic costs. The second major effort of our study is the formulation of the decentralized control design as a constrained optimal output feedback problem. In this framework, we have developed necessary conditions for existence of an optimal decentralized control law, derived two numerical algorithms for the computation of the optimal decentralized control, and proved the convergence of the decentralized algorithm under suitable conditions. Finally, convergence characteristics of the two algorithms are compared using a tenth order model of a jet aircraft example involving the joint design of flight and engine control subsystems.

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## Foreword

The research reported herein was accomplished for the United States Air Force by Bolt Beranek and Newman Inc. (BBN), under Contract F33615-82-C-3612 and by Information and Control Systems Inc. (ICS) under Subcontract 28930. The Air Force Wright Aeronautical Laboratory's Flight Dynamics Laboratory is the program sponsor.

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## Executive Summary

The purpose of this study is to develop a computational technique for designing various subsystem controllers of a physical system in a coordinated manner, e.g. the design of engine and flight control subsystem controllers on an aircraft. In a coordinated design, the controllers are designed jointly so that the designed subsystems cooperate with each other, thus, minimizing potential subsystem interactions which can lead to performance degradation such as increased fuel expenditure.

The major results of our study are as follows. First, we have found conditions on the subsystem controller objectives guaranteeing the non-interaction between these performance indices. Second, we have formulated the design as a constrained output feedback problem and found sufficient conditions which guarantee the existence of an optimal control law. The importance of these conditions is that the existence of a solution can be determined at the outset of the design process from the known subsystem and objective function parameters.

Next, we have developed an iterative numerical algorithm for computing the optimal control law and proved that that algorithm converges to an optimal solution under the sufficiency conditions ensuring the existence of an optimal solution. This convergence results is important since the sufficiency conditions include a large class of practical design problems.

Finally, the developed algorithm is used to design an integrated flight and engine control system based on the tenth order linearized longitudinal dynamics of a fighter

aircraft. Simulation results show that the coordinated design, in which subsystem controllers use only the subsystem measurements for feedback, has a performance level comparable to that of the full state feedback centralized controller.

## 1. INTRODUCTION

This basic research study has been concerned with the investigation of coordinated control systems design for improved fuel efficiency. We believe that the coordinated control algorithm developed here would be useful not only in improving fuel efficiency, but also in optimizing the coordination between the various control systems on a given aircraft for improved system performance.

Coordinated control encompasses the general problem of control subsystems design, in which controllers corresponding to different subsystems work in coordination with each other. In these systems, there are a multiple number of control subsystems, each with a different control objective and each with a different controller structure driven by a different subset of the plant measurements. In a coordinated design, controller performance objectives are jointly optimized over the parameters of their structures so that the designed subsystems cooperate with each other. Thus, potential interactions between subsystems which can lead to performance degradation (such as increased fuel expenditure) are eliminated in a coordinated design.

The problem of control subsystems design, in which controllers corresponding to different subsystems work in coordination with each other, falls into the general area of decentralized control. In our study, we have considered the following class of decentralized control problems. The plant dynamics are described by a linear discrete-time stochastic time-invariant system. There are a multiple number of control subsystems, each with a different control objective and each with a different

linear noisy measurement subset of the plant state. The controller objective of each subsystem is modelled by a quadratic cost functional. Moreover, the complexity of each control subsystem is constrained such that each controller is restricted to be a linear dynamic system driven by the measurements available to that controller.

Our study has two major thrusts. The first one is concerned with the selection of different controller cost functionals such that these performance indices do not conflict with each other. In particular, we have defined the notion of noninteraction for quadratic cost functionals and found necessary and sufficient conditions for noninteraction between quadratic costs.

The second major effort of our study is the formulation of the decentralized control design as a constrained optimal output feedback problem. In this framework, we have developed sufficient conditions for the existence of an optimal decentralized control law, derived two numerical algorithms for the computation of the optimal decentralized control, and proved the convergence of the decentralized algorithm under suitable conditions. Finally, convergence characteristics of the two algorithms are compared using a tenth order model of a jet aircraft example involving the joint design of flight and engine control subsystems.

The developed coordinated control algorithms are applicable to quite a number of relevant problems in the context of aircraft control. For instance, the proposed formulation encompasses the following types of control subsystems design problems: simultaneous design of a structure mode control system [15]-[16] and a stability augmentation system (SAS) [14], joint design of an autothrottle [20] and an altitude-

hold system; joint consideration of a SAS design and pilot dynamics. In fact, all control subsystems on a given aircraft can be considered simultaneously within the scope of the proposed study. Furthermore, the proposed formulation can handle the problem of designing different control loops in a given aircraft control subsystem. For instance, the problem of joint design of pitch [18], roll and yaw [19] control loops in a given SAS can be resolved within this framework.

We believe that the decentralized control law algorithms which we have developed here make an important contribution to decentralized control. As remarked by Sandell, et.al. in [22]: "Perhaps if our insistence that the solution be optimal with respect to the very wide class of all functions of past observations is relaxed, a solution that is computable, implementable, and robust can be obtained". That is, fixed controller structure approach taken in our study is certainly feasible and may well be the appropriate extension of the LQG methodology to decentralized control. It is precisely this goal of developing decentralized control laws which are computable, implementable, robust and, of course, stable, and perform their objective satisfactorily that our study addresses.

The organization of the report is as follows: background information about aircraft fuel efficiency is given in the next section. Section 1.2 contains a specific example showing the potential performance degradation which can occur when subsystem controllers do not cooperate with each other. Technical results are presented in Chapter 2. An overview of decentralized control theory aspects having the most relevance on cooperating control subsystems design is given in Section 2.1. The mathematical description of the class of problems considered in our study is

developed in Section 2.2. Sections 2.3-2.5 contain the results for nonconflicting multiple quadratic cost functional selection. A constrained output feedback problem equivalent to the posed decentralized control problem is given in Section 2.6. Section 2.7 contains the existence results for the optimal decentralized control. Incremental cost and necessary conditions of optimality are given in the next section. Section 2.9 contains the numerical algorithms for the computation of the optimal decentralized control law. Convergence results are presented in the next section. Simulation results for an example problem concerning the integrated design of an engine and flight path control subsystems using the developed algorithms are given in the first section of Chapter 3. The relevance of using nonlinear decentralized control laws for this example is discussed in the next section. Finally, Chapter 4 contains the conclusions and recommendations for future study.

For a quick overview of the report, we recommend that the reader read Sections 1.1, 1.2, 2.1, 2.2, introductory paragraphs of Sections 2.3, 2.6, 2.7, 2.8, 2.9, and 2.10, and Chapters 3 and 4. Sections 1.1 and 1.2 may be skipped if motivation for the problem is not needed. Similarly, Sections 2.1 and 2.2 may be skipped if the reader is familiar with prior work in this area.

## 1.1 Background

Escalating costs of fuel during the last decade have made it mandatory that the aerospace industry should strive to develop feasible means for fuel conservation [1]. Motivated by the incentives associated with reduced fuel consumption, such as increased profit margins, commercial airlines have developed computerized flight planning systems to optimize direct operating costs [2]. Those computerized flight

planning systems are used to find, for a given departure and destination pair, the necessary minimum fuel, the optimum flight path checkpoint sequence, the optimum flight altitude and speed for given takeoff gross weight, atmospheric wind, and temperature conditions. The minimum fuel computations are important since approximately one-tenth of the extra fuel carried on a medium distance flight is wasted to carry this extra fuel. Since the fuel considerations usually dictate a lower airspeed and, hence, increased trip time, the commercial carriers usually opt to minimize total direct operating costs which include personnel expenses in addition to fuel expenditure. Optimum flight altitude profile is implemented as a sequence of step changes in altitude during cruise within the ATC constraints. These flight planning systems are based on dynamic programming optimization techniques, and utilize the aircraft performance data tables and standard operating procedures, such as constant Mach or Long Range Cruise.

Aircraft manufacturers have also started to look at the practicality of implementing closed-loop energy management systems for jet transports [3]. These efforts involve on-board computation of a best energy operating state and the execution of autopilot and autothrottle commands to achieve this performance. These methods are based on the results achieved through mostly government-supported research over the last two decades [4]-[11]. In most of these schemes, aircraft point mass equations of motion are utilized to compute the optimum flight trajectory using numerical optimization methods. Since the direct approach usually involves time consuming iterative procedures such as steepest descent, quasilinearization, etc., simplified criterion such as specific energy state is used for minimization.

Specific energy state is the sum of the aircraft kinetic and potential energies divided by the aircraft weight. For instance, an optimal climb profile is obtained by maximizing the rate of change of specific energy per unit fuel weight [6]. Although the closed-loop optimization technique for climb and descent was determined infeasible for current commercial transports [3], a new closed-loop cruise algorithm, used in conjunction with an airspeed-hold-mode autothrottle and an altitude-hold-mode autopilot, was proven to be feasible. Since a fuel savings of about 3% over conventional cruise procedures is possible with these new cruise algorithms, they will certainly be implemented in the next generation commercial aircraft.

For military aircraft, fuel efficiency is desired not only to reduce operational costs but also to obtain increased mission range, heavier payload capability for a specific aircraft, and more training missions for a given fuel budget [12]-[13]. Since the current annual fuel usage of the U.S. Air Force is approximately 4 billion gallons [13], even a one-percent savings in the fuel consumption would provide a substantial benefit to the Air Force.

We believe that the flight path optimization theory is adequately developed. The current state-of-the-art provides an adequate basis for the industry to proceed with the implementation of these concepts [3]. The potential benefit to fuel efficiency which can be derived from the coordination of various aircraft control system designs have not been explored. In the next section, we will give a specific example highlighting an undesirable interaction between two control subsystems.

## **1.2 Two Uncooperating Subsystems -- An Example**

The commonly used approach of designing aircraft control subsystems independently from each other can result in an overall system where the subsystems do not cooperate in the achievement of an overall goal, but tend to oppose each other. The following specific example of two uncooperating control subsystems is taken from the currently ongoing flight tests of the Digital Integrated Automatic Landing System (DIALS) [17] at NASA's Langley Research Center.

During the first minute of this flight test, the aircraft is controlled by an altitude hold control system coupled with an autothrottle system previously designed using a conventional classical approach. The two subsystems were designed to meet different objectives, the goal of the autothrottle being to maintain the selected airspeed, the goal of the altitude hold system being to maintain a constant altitude or zero sink rate. Figure 1 shows the thrust (engine pressure ratio), and sink rate history (from the flight charts). As seen in Figure 1, the sink rate oscillates with a peak-to-peak deviation larger than 25 ft/sec, instead of remaining near zero. Similarly, the thrust is seen to oscillate with a peak-to-peak deviation of 14,000 lbs. in the autothrottle/altitude hold mode.

This behavior of the autothrottle/altitude hold mode can be traced to the fact that the coupling between the two subsystems in terms of the airspeed and the vertical variables, such as pitch angle and sink rate through the aircraft dynamics, was not sufficiently accounted for. Thus, when the aircraft maintains a constant altitude, the oscillatory behavior does not occur. However, if disturbances reduce the

aircraft's altitude, the altitude hold system pitches the aircraft up to obtain more lift, with the added effect of also increasing the drag. The autothrottle then notices the decreasing speed and commands a higher throttle which produces an increasing thrust with a lag (due to the slow engine dynamics). Thus, when the aircraft reaches its desired altitude, the thrust is considerably higher than its required trim value and overshoots in altitude as well as speed. The altitude hold system pitches the aircraft down to get back onto its desired altitude, which further increases the speed, and the cycle continues. Clearly the two control subsystems do not cooperate when both are active, whereas each one performs its task adequately without the other. This behavior is mainly due to the design approach where each subsystem is designed with little consideration of other subsystems.

At approximately one minute into the flight test, a centralized automatic landing system is engaged. As can be seen from Figure 1, this centralized control system quickly stabilizes the thrust to a stable trim value while performing a glide slope capture maneuver. The centralized controller is an integrated control system using full state feedback based on modern control theory principles. As this simple example demonstrates, an integrated system design approach is often necessary to ensure the cooperation of various subsystems to achieve an overall objective. In this case, the penalty on fuel efficiency arising from the lack of cooperation between subsystems is clear. On the other hand, the interaction between the subsystems could also have an indirect but quantifiable impact on total fuel consumption. While the centralized control system using full state feedback given in this example achieves the overall objective, the design of centralized flight control systems is often impractical. Moreover, separate control subsystem structures are more desirable in military

applications due to reliability and survivability considerations. A method to design control subsystems which act in cooperation can be obtained by using decentralized control theory. In the first section of the next chapter, we shall give a technical review of the relevant theory.

## 2. MATHEMATICAL DEVELOPMENT

This chapter contains our findings concerning the design of a set of coordinating subsystem controllers. We start with an overview of the related prior work in decentralized control.

### 2.1 Review of Related Prior Work

During the last three decades, a large body of system and control theory developments and advances in computer technology have enabled control and system engineers to develop generalized algorithms which are directly applicable to complex system design problems. The major developments have almost invariably considered systems with a centralized sensor and control structure; i.e., the system is controlled by a single control unit having access to all the available information obtained by all the sensors. The results obtained have shown that, in many cases, linear feedback of state estimates is sufficient to provide satisfactory system behavior, and it is, in fact, optimal for the LQG problem. Pole placement in linear systems can usually be achieved with linear feedback. Efficient numerical algorithms which use the system model are available to determine control system parameter values. These algorithms provide excellent starting points for the system designer in pursuit of a working centralized controller for a complex system. However, these developments have left the problems associated with decentralized systems largely unsolved.

While systems with a decentralized sensor network where a decentralized control structure is desirable are common, (e.g., electric power systems, transportation systems, economic systems, etc.), a decentralized control structure may be sought for

reasons other than a decentralized sensor system. Examples can be found in aircraft control systems, where the natural modes of the system are widely separated (e.g., the structural modes and the phugoid mode) and where it is desirable to use one subsystem with many others depending on the task performed. For example, one inner loop system is usually used with different outer loop guidance systems that depend on the path to be followed.

Recently, some attention has been focused on questions associated with decentralized systems [22]. Although some results have been obtained, the main developments in decentralized system theory have been of a negative nature. The separation of estimation and control is not optimal for linear decentralized control systems with quadratic cost and Gaussian disturbances. In fact, the optimal control is not necessarily a linear feedback law, as noted by Witsenhausen [37]; in general, the existence of an optimal law is not guaranteed [21] for decentralized systems, and the optimal linear law can be of infinite dimensions, and hence not realizable [38]. Questions as to the importance of "signalling", "second guessing" and the extent of "cross-communication" among decentralized controllers (which do not arise in centralized systems), remain to be answered.

As discussed in the recent survey paper [22], the work on decentralized control theory can be divided into the following broad categories:

- o stability methods.
- o model approximation methods.
- o optimal decentralized feedback.
- o optimal decentralized filtering.

- o multilevel control systems.

Stability methods usually consider the deterministic case. The basic objective of this approach is to determine if a system can be stabilized by a decentralized controller with a specific fixed structure and to obtain algorithms for computing stabilizing feedback gains. Both Lyapunov [23]-[24] and input-output [25]-[26] methods have been applied to the stability determination problems in interconnected systems. These approaches use bounds on the interconnection effects and essentially treat them as disturbances.

Stabilizability and controllability of decentralized systems have been studied in [27]-[29]. An important result on the stabilizability of decentralized systems has been derived in [28]. Namely, it is shown that a necessary and sufficient condition for stabilizability is that the fixed modes of a decentralized system should have negative real parts. Furthermore, the computational requirements for determining the fixed modes consist of finding the eigenvalues of large matrices, for which efficient subroutines are available. Pole placement methods have also been investigated [30]-[31]. These methods tend to load a single local controller by giving it most of the task of controlling the system. Furthermore, computational schemes associated with these methods are quite complex.

Model simplification techniques for decentralized systems fall into two categories: aggregation methods and perturbation techniques. Aggregation concepts [32]-[33] generalize the standard engineering decisions used in simplifying large scale systems. Rather than completely ignoring certain parts of the dynamics, aggregation methods

try to add the effect of neglected modes onto the simplified model. Singular perturbation theory has also been used effectively in model reduction problems [34]–[36] for systems where the natural modes are widely separated.

Optimal decentralized control laws maximizing a quadratic cost function have also been widely studied. The celebrated example of Witsenhausen [37] demonstrate that the solution with Gaussian disturbances is not linear and the separation theorem does not hold in stochastic decentralized control problems. It was further shown that the optimal linear decentralized law, for a rather simple discrete system [38], requires an infinite dimensional controller. Since the restriction of control laws that are arbitrary linear functions of past observations do not necessarily result in an implementable solution, the next attempt has been to restrict the controllers to be of a linear fixed-order structure and to optimize over the parameters of this structure. A stochastic suboptimal design is considered in [39], where the measurements of each controller drive a local filter which provides unbiased estimates of the complete system and the local control is a linear feedback of the local estimate. Clearly, the computational requirements arising from the use of a large filter at every controller are very demanding.

A similar approach is to pose the decentralized control problem as a constrained output feedback problem [40]–[41]. For instance, the method in [41] requires the solution of a nonlinear algebraic matrix equation in order to compute the parameters of the constrained decentralized compensators. We agree with the view stated in [22] that the fixed structure approach is feasible and is the most promising extension of the LQG methodology to decentralized systems. The main difficulty with this approach

is that satisfactory algorithms to solve the problem are at best rare, and require lengthy searches at every iteration, whenever convergent [42]–[47]. A review and comparison of these algorithms can be found in [47].

In decentralized filtering, extensions of the standard LQG estimation algorithms to systems with fixed information exchange patterns have been studied. For instance, the formulation in [48] is a constrained nonlinear programming problem with the filter, and control gains which are not dictated to be zero by the decentralized structure, taken as independent variables. Although this method has been successful in traffic flow applications where the coupling is mostly between the nearest neighbors, in general, this technique neither assures closed-loop stability nor convergence to a global minimum. In contrast to this joint optimization approach to designing the local estimators, in [50], the authors have developed locally unbiased filters by assuming that complete measurements of the interactions from other subsystems into the local subsystem are available to the local decision maker along with at least one local state measurement. Their result is an uncoupled set of filtering algorithms for the local estimators obtained through the application of the matrix minimum principle. The main disadvantage of this technique is the requirement of complete measurements of the interaction between the subsystems. Another class of decentralized estimators results from the application of perturbation techniques. In the case of weak coupling between the subsystems [51], suboptimal decentralized estimators have been obtained by extending separate bias estimation technique to the case when the bias states are dynamically coupled [52]–[53] with the system states and then using perturbation methods to obtain the approximate filters.

We also note that the solutions to the decentralized LQG control problem have also been obtained [54]-[55]. Computation of these optimal Nash strategies involve the solution of coupled Riccati matrix equations [55]. There has also been some applications of these results to aircraft control systems. For instance, the computed SAS and pilot control laws in [56] are, in fact, the optimal Nash strategies in the space of linear full state feedback controls. The major drawback of this approach is, of course, the requirement for full state feedback.

Multilevel control structures for decentralized control have also been investigated [57]-[59]. The optimal Stackelberg strategies for these systems are derived such that one decision maker, called the leader, announces his feedback strategy before the other decision makers, called followers, select their control strategies. Optimal Stackelberg strategies for LQ games can be computed by solving coupled Riccati matrix equations [58]-[59]. Other multilevel controller structures involving a high level coordinator manipulating lower order controllers have been explored in [60]. The applicability of multilevel controller structures to aircraft control problems remains to be established.

In summary, we believe that decentralized stochastic control and estimation methods using linear and fixed controller and estimator structures appears to be the most feasible approach for aircraft applications. The fact that this formulation does not address the question of what structure should be assumed may be considered a drawback in general decentralized system applications. However, this aspect would not be a problem in flight control system applications due to the large known body of knowledge about aircraft control subsystems. In the next section, we shall present

the mathematical description of the class of problems which will be considered in our study.

## 2.2 Formulation of the Optimal Decentralized Control Problem

Since the optimal decentralized control law for a linear plant, quadratic cost and Gaussian statistics can be nonlinear, and the optimal linear control law can be of infinite dimensions [38], hence not implementable, the next natural class of control laws that can be considered is the class of implementable linear decentralized control laws, i.e., a decentralized control law where each subsystem controller is constrained to be of finite order. The order of each subcontroller is considered a design parameter specified by the designer. It is assumed that the plant has been linearized, and that a stochastic sampled-data formulation has been selected for digital implementation [61]. Thus, we consider the following discrete-time stochastic linear plant

$$x_e(k+1) = A_e x_e(k) + \sum_{i=1}^p B_i u_i(k) + w_e(k), \quad k \geq 0. \quad (1)$$

where  $x_e(k)$  is the  $n$ -vector state,  $u_i(k)$  is the  $r_i$ -vector control for the  $i^{\text{th}}$  subsystem, and  $w_e(k)$  is a zero-mean white noise sequence, where (denoting the expectation operator by  $E$ ).

$$E(w_e(k)) = 0 \text{ and } E(x_e(0)) = 0 \quad (2)$$

$$E(w_e(k) w_e(j)) = W_e \delta_{kj} \quad (3)$$

$$E(w_e(k) x_e(0)) = 0 \quad (4)$$

$$E(x_e(0) x_e(0)) = S_e \quad (5)$$

In usual applications of decentralized control, the plant,  $A_0$ , contains a number of loosely coupled subsystems controlled by the subsystem controls,  $u_i(k)$ . Each subsystem control,  $u_i(k)$ , is constrained to use only a corresponding set of measurements,  $y_i(k)$ , accessible to that controller. That is, we consider the following nonclassical information pattern:

$$y_i(k) = C_i x(k) + v_i(k) \quad 1 \leq i \leq p \quad (6)$$

where  $y_i(k)$  is a  $m_i$ -vector measurement available to the  $i$ 'th subsystem controller,  $v_i(k)$  is a zero-mean white measurement noise sequence, and

$$E(v_i(k)) = 0 \quad (7)$$

$$E(v_i(k) v_j(k)) = V_{ij} \delta_{kj} \quad (8)$$

$$E(v_i(k) x_o(k)) = 0 \quad (9)$$

$$E(v_i(k) w_o(k)) = 0 \quad (10)$$

As seen from above, it is assumed that the plant process noise sequence, measurement noise sequence, and plant initial condition are mutually uncorrelated. It should be pointed out that the initial condition having zero mean, and the measurement and plant noises being uncorrelated are conditions which can be easily relaxed at the expense of slightly more complicated expressions in the following.

The measurement vectors,  $y_i(k)$ , are not restricted to contain different components, and may contain common variables. That is, the formulation encompasses the case in which the state measurements are divided into subsets with nonempty intersections. The significance of allowing the same variable in the feedback of different subcontrollers is that coordination between subsystem controllers can be

greatly enhanced by selecting appropriate common feedback components. On the other hand, the case inwhich there is no correlation between the measurement noises corresponding to different subsystems is included in this formulation as well.

Similarly, the formulation presented here allows the use of the same control component in more than one subsystem controller, when this is desirable. To illustrate this point, consider the case where the aileron is used to achieve the two goals of flutter suppression and flight path control. The aileron would then be included in both the flutter suppression subcontroller and the flight control system. The flutter suppression subcontroller would feed back accelerometer measurements placed at appropriate locations on the wing, and would have high frequency content. Whereas the flight path subcontroller would feed back flight path variables and have relatively lower frequency content. Thus, the total aileron command would contain both high and low frequency components achieving both flutter suppression and flight control objectives in a coordinated manner. However, if desired, the flutter suppression subcontroller commands could be computed in a small microprocessor located at or near the wing, rather than a central computer.

As mentioned earlier, it is desirable to obtain subsystem controllers which are of finite order, so that their implementation is possible and simple. Furthermore, to obtain a decentralized control structure it is necessary for  $u_i(k)$  to depend only on past and present measurements of the selected measurement vector  $y_i(k)$ . Therefore, we consider the class of controllers where each subsystem controller is constrained to be a finite order linear dynamic system of the form:

$$z_i(k+1) = E_i z_i(k) + H_i y_i(k) \quad (11)$$

$$u_i(k) = G_i z_i(k) + F_i y_i(k) \quad 1 \leq i \leq p \quad (12)$$

where  $z_i(k)$  is the  $q_i$ -vector representing the state of the dynamic compensator for the  $i^{th}$  subcontroller driven by the appropriate measurement vector. The order of the dynamic compensator,  $q_i$ , for each subsystem is a design specification to be selected by the designer. The controller is formulated to be time invariant; i.e.,  $E_i$ ,  $H_i$ ,  $G_i$ ,  $F_i$  do not vary with time. These parameters which determine the dynamic compensation and control commands are the parameters for optimization.

As each subcontroller has a somewhat different task, a cost or objective function is selected to describe each subcontroller task, in the form

$$J_i = \lim_{N \rightarrow \infty} J_{iN} \quad (13)$$

$$J_{iN} = \frac{1}{2(N+1)} E \sum_{k=0}^N x_o(k+1)^T Q_i x_o(k+1) + u_i(k)^T R_i u_i(k) \quad (14)$$

Note that  $J_{iN}$  is the cost criterion over a finite period of time, while  $J_i$  is the average steady-state cost, which is the desired criterion. Note that  $J_{iN}$ , hence  $J_i$ , depend not only on the  $i^{th}$  subcontroller parameters, but on all the subcontrollers. That is, due to the coupling between the subsystems of the plant, the optimal strategy for one subsystem controller depends, even if slightly, on the other subsystem controller strategies. As this coupling is incorporated in the plant equations and the cost functions, the optimization drives the subsystem controllers to cooperate to the extent possible. For instance, in aircraft flight control applications, the autothrottle subsystem objective may be to null the airspeed error using a minimum of throttle activity, while the objective of the pitch stability augmentation subsystem would be to provide adequate damping with low elevator activity. However, the overall design

objective is to meet all the subsystem objectives which must operate simultaneously, so that each subsystem enhances the operation of the others rather than degrade it. Hence, the cost or objective function for the total decentralized control law is formulated as a weighted linear combination of the objectives of each subcontroller:

$$J = \lim_{N \rightarrow \infty} J_N \quad (15)$$

$$J_N = \sum_{i=1}^p a_i J_{iN} \quad a_i > 0 \quad 1 \leq i \leq p \quad (16)$$

where  $a_i$ 's are selected according to the relative importance of subsystem objectives.

The optimal decentralized control problem can now be posed as the problem of finding the parameters,  $\{E_i^*, H_i^*, G_i^*, F_i^*, 1 \leq i \leq p\}$  of a decentralized control law of the form (11) and (12), which stabilize the closed-loop system and minimize the cost  $J$  in (15), subject to the constraints imposed by the system dynamics and the information pattern given in (1) - (10).

It is important to note that in this formulation, when an optimal control exists, it will be implementable, and will stabilize the closed-loop system. The optimization problem will be posed with more mathematical rigor in Section 2.4. Before that however, we will present our results on nonconflicting, quadratic multicriteria selection in the next section.

### 2.3 Nonconflicting Multi-Criteria Selection

The cost functionals, eq. 13, for various subsystem controllers will, in general, have conflicting requirements. For simplicity, the desired set points for subsystems

are taken to be null vectors in eq. 13. In practice, the desired set point for some subsystem may be a specific trajectory or the output of an ideal model to be followed [62].

In this case, the subsystem objectives may be conflicting with each other in the sense that while one subsystem controller is trying to force a certain system state to be small for a smooth response, another subsystem controller may well be encouraging the same state variable to be large in order to follow a certain trajectory. This difficulty may be overcome if the relative importance of  $i$ 'th objective relative to the other objectives can be specified so that other cost functionals can be modified for noninteraction.

In this section, we define the notion of noninteraction between quadratic cost functionals and give a necessary and sufficient condition for it. Our results are based on the following theorem from matrix theory (See Theorem 12 on pg. 10 in [65]):

Theorem 1: Let  $H$  be a linear transformation on  $\mathbb{R}^n$  to  $\mathbb{R}^m$ , i.e.  $H: \mathbb{R}^n \rightarrow \mathbb{R}^m$ . If  $H^\#$  is the pseudo-inverse<sup>1</sup> of  $H$ , then we have the following direct sum decompositions for  $\mathbb{R}^n$  and  $\mathbb{R}^m$ :

$$\mathbb{R}^n = \tilde{N}(H) \oplus \tilde{R}(H^\# H) \quad (17)$$

$$\mathbb{R}^m = \tilde{R}(H) \oplus \tilde{N}(HH^\#) \quad (18)$$

---

<sup>1</sup>Actually, these results hold for the weaker generalized inverse. However, we will need the properties of the pseudo-inverse in the following.

where  $\tilde{N}$  and  $\tilde{R}$  denotes null space and range of the associated operator.

For every vector  $x$  in  $R^n$  and  $y$  in  $R^m$ , the representations for the direct sums in the theorem above are given by:

$$x = [I - H^{\#}H]x + H^{\#}Hx \quad \text{for } x \text{ in } R^n \quad (19)$$

$$y = HH^{\#}y + [I - HH^{\#}]y \quad \text{for } y \text{ in } R^m \quad (20)$$

where  $H^{\#}$  is the pseudo-inverse of  $H$ . The first term in eq. (19) above is the projection of  $x$  onto the null space of  $H$  and the second term is the corresponding orthogonal complement (i.e., the projection onto the range of  $H^{\#}$ ). Let us verify these assertions. Starting with the first term:

$$H [I - H^{\#}H]x = [H - HH^{\#}H]x = [H - H]x = 0 \quad (21)$$

Eq. 21 follows since, by definition, we have  $H = HH^{\#}H$ . Therefore,  $[I - H^{\#}H]x$  is in the null space of  $H$ . Now, taking the inner product of the two terms in eq. 19, we have

$$\langle [I - H^{\#}H]x, H^{\#}Hx \rangle = x' [H^{\#}H - (H^{\#}H)'H^{\#}H]x \quad (22)$$

Since we have  $(H^{\#}H)' = H^{\#}H$  from the definition of the pseudo-inverse, eq. 21 becomes:

$$\langle [I - H^{\#}H]x, H^{\#}Hx \rangle = x' [H^{\#}H - H^{\#}(HH^{\#}H)]x = 0 \quad (23)$$

so that the second term is indeed the orthogonal complement. Representation for  $y$  in eq. 20 can similarly be proved.

Now, returning back to subsystem controller functionals described by eqs. 13-14.

we will now show how conflicting requirements can be taken out. For instance, suppose the designer decides that the first controller objective is relatively more important than the objective of the second subsystem. Hence, it is desired to constrain the second subsystem cost functional, so that the objective of the second controller is not in conflict with that of the first one. Denoting the square root of  $Q_1$  in eq. 14 by  $O_1 = H_1^T H_1$ , if we constrain the weighting matrix  $Q_2$  of the second controller to be of the form:

$$Q_2 = [I - H_1^T H_1]^\frac{1}{2} H^T H [I - H_1^T H_1]^\frac{1}{2} \quad (24)$$

where  $H$  is arbitrary, then the second objective would not be in conflict with the first one. This is because the second controller would penalize only those state excursions which are in the null space of  $H_1$ . This follows from the discussion following Theorem 1 since the vector  $[I - H_1^T H_1]x$  for arbitrary  $x$  in  $R^n$  will be in the null space of  $H_1$ .

We would like to be able to choose the arbitrary matrix  $H$  in eq. 24 such that

$$[I - H_1^T H_1]^\frac{1}{2} H^T H [I - H_1^T H_1]^\frac{1}{2} = H_2^T H_2 \quad (25)$$

or, what is the same,

$$H [I - H_1^T H_1]^\frac{1}{2} = H_2 \quad (26)$$

If we could choose an  $H$  such that eq. 26 is identically satisfied, then the second controller's cost functional need not be modified. However, in general, there would not be a solution of this equation for  $H$ . The best approximate solution in the least squares sense would be given by

$$H^* = H_2 [I - H_1^T H_1]^\frac{1}{2} \quad (27)$$

We will now show that the generalized inverse of  $[I - H_1^T H_1]$  is equal to itself. To show this, we will need the following facts:

Lemma 1: If  $P$  is an idempotent [63] matrix, then  $I - P$  is also idempotent.

Proof: Recall that  $P$  is idempotent if  $P^2 = P$ . So,  $(I - P)$   
 $(I - P) = I - P - P + P^2 = I - P - P + P = I - P$ .

Lemma 2: If a real symmetric matrix  $P$  is idempotent, then  $P^T = P$ .

Proof: This follows since  $PPP = PP = P$  and  $(PP)' = P' = P = PP$ .

We can now find the generalized inverse of  $I - H^T H$  using these results above.

Theorem 1: If  $H$  is an arbitrary matrix, then  $[I - H^T H]^T = I - H^T H$ .

Proof: Since  $(H^T H)H^T H = H^T(HH^T H) = H^T H$ , the matrix  $H^T H$  is idempotent. It then follows by Lemma 1 that  $I - H^T H$  is idempotent. Since  $I - H^T H$  is also symmetric, by Lemma 2, we have  $[I - H^T H]^T = [I - H^T H]$ .

Therefore, the best approximate solution  $H^*$  given by eq. (27) becomes

$$H^* = H_2 [I - H_1^T H_1] \quad (28)$$

Substituting the expression for  $H^*$  into eq. 24 and simplifying, we get for the

second cost functional

$$Q_2 = [I - H_1^T H_1] H_2^T H_2 [I - H_1^T H_1] \quad (29)$$

That is, the second cost functional when modified as above would not be in conflict with the first one. These results generalize those on pp. 248-250 in [64]. Note that if  $H_2[I - H_1^T H_1] = H_2$ , then  $Q_2$  would be nonconflicting with  $Q_1$  without any modification. Since this condition would imply  $H_2 H_1^T H_1 = 0$ , it suggests the following definition for two noninteracting cost functionals.

Definition 1: Let  $Q_1 = H_1^T H_1$  and  $Q_2 = H_2^T H_2$  be positive semi-definite matrices. The quadratic forms  $x^T Q_1 x$  and  $x^T Q_2 x$  are noninteracting if  $H_2 H_1^T H_1 = 0$ .

We will next show that the definition is symmetric. That is, we can interchange the subscripts 1 and 2. Before that, however, we need the following fact:

Lemma 3: If  $H$  is an arbitrary matrix, then  $H^T = H^T H H^T$ .

Proof. Since  $H = H H^T H$ , we have  $H^T = [H(H^T H)]^T = (H^T H)^T H^T$ . From the definition of the pseudo-inverse, we have  $(H^T H)^T = H^T H$  so that  $H^T = H^T H H^T$ .

Theorem 2: Let  $Q_1 = H_1^T H_1$  and  $Q_2 = H_2^T H_2$  be positive semi-definite matrices. The quadratic forms  $x^T Q_1 x$  and  $x^T Q_2 x$  are noninteracting if and only if  $H_1 H_2^T H_1 = 0$ .

Proof. Let  $Q_1$  and  $Q_2$  be noninteracting. Then, we have  $H_2 H_1^T H_1 = 0$ . Post multiplying by  $H_1^T$ , we get  $H_2 (H_1^T H_1 H_1^T) = 0$ . Since  $H_1^T = H_1^T H_1 H_1^T$ , by Lemma 3, this implies

$H_2 H_1' = 0$ . This would imply  $H_1 H_2' = 0$ . Postmultiplying by  $(H_2')' H_2' H_2$  and using the fact  $H_2' (H_2')' H_2 = H_2$ , we get  $H_1 H_2' H_2 = 0$ . Starting from  $H_1 H_2' H_2 = 0$ , it can similarly be shown that this would imply  $H_2 H_1' H_1 = 0$ .

A computational check for noninteraction is given by the following:

Theorem 3: Let  $Q_1 = H_1' H_1$  and  $Q_2 = H_2' H_2$  be positive semi-definite matrices. Two quadratic forms  $x' Q_1 x$  and  $x' Q_2 x$  are noninteracting if and only if  $H_2 H_1' = 0$ .

Proof: Let  $Q_1$  and  $Q_2$  be noninteracting. We then have  $H_2 H_1' H_1 = 0$ . Postmultiplying by  $H_1'$ , we get  $H_2 (H_1' H_1 H_1') = 0$ . Since  $H_1' H_1 H_1' = H_1'$ , this implies  $H_2 H_1' = 0$ .

Now let  $H_2 H_1' = 0$ . Postmultiplying by  $(H_1')' H_1'$ , we get  $H_2 H_1' (H_1')' H_1' = 0$ . Since  $H_1' (H_1')' H_1' = H_1'$ , this implies that  $H_2 H_1' H_1 = 0$ .

By using the necessary and sufficient condition for noninteraction given by Theorem 3 above, it can easily be determined whether two quadratic cost functionals are non-interacting or not. The following example outlines the procedure.

Example 1: Let  $Q_1 = H_1' H_1$  and  $H_2' H_2 = Q_2$  be two weighting matrices given by

$$H_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \quad \text{and} \quad H_2 = [h_1 \ h_2 \ h_3]$$

So  $H_1 H_2' = 0$  implies that  $h_1 + h_2 = 0$  and  $h_2 - h_3 = 0$ . Therefore, any  $H_2 = h[1 \ -1 \ -1]$

would produce a non-conflicting weighting matrix.

#### 2.4 Extension to Multiple Subsystems

Noninteracting quadratic cost selection procedure can be extended to multiple cost functionals, if the relative importance of controller subsets with respect to each other can be specified by the designer. For instance, the objectives of subsystems subset  $(S_1, S_2)$  may be more important than that of the third subsystem. On the other hand, the objectives of subsystems subset  $(S_3, S_4, S_5)$  may be considered to be more important than those of the subset  $(S_6, S_7)$ . In this fashion, a hierarchy of importance levels can be specified for subsystem subsets. Then the quadratic costs can be chosen in the following manner such that the subsystems criteria would not be conflicting with each other.

In the two subsystem case, say  $i$ 'th and  $j$ 'th subsystems, the result is obtained by writing the state space as a sum of the range of  $H_i$  and its orthogonal complement and constraining the  $j$ 'th controller cost functional onto this orthogonal complement. In the case of multiple subsystems, for instance in the example above, third subsystem cost is constrained onto the orthogonal complement of the range  $H_{12}$  where:

$$H_{12} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \quad (30)$$

In this case, the third subsystem quadratic form would then be given by (referring back to eq. 24).

$$Q_3 = (I - H_{12}^T H_{12})^T H^T H (I - H_{12}^T H_{12}) \quad (31)$$

where  $H$  is arbitrarily selected.

## 2.5 Weaker Nonconflicting Criteria

Nonconflicting quadratic criteria selection procedure can be modified to have weaker restrictions on the subsystem cost functionals. For instance, in the case of subsystems one and two, it is possible to restrict the second controller functional either onto the null space of  $H_1$ , or onto the orthogonal complement of the controllability subspace of the first controller. To illustrate the procedure, consider the controllability subspace  $\Gamma_1$  of the first controller, i.e.,

$$\Gamma_1 = [B_1, AB_1, A^2B_1, \dots, A^{n-1}B_1] \quad (32)$$

From the discussion following Theorem 1, the transformation  $\Gamma_1$  is associated with the following direct sum decomposition of the state space  $\mathbb{R}^n$ :

$$\mathbb{R}^n = \tilde{R}(\Gamma_1) \oplus \tilde{N}(\Gamma_1) \quad (33)$$

So that the second controller cost functional can be restricted to penalize the state excursions either in the null space of  $H_1$ , or in the null space of  $\Gamma_1\Gamma_1^\dagger$ . Since the representation for this direct sum decomposition is given by eq. 20, i.e.,

$$x = \Gamma_1\Gamma_1^\dagger x + (I - \Gamma_1\Gamma_1^\dagger)x \quad (34)$$

Therefore, the second subsystem cost functional can be constrained to be

$$\tilde{Q}_2 = Q_2 + (I - \Gamma_1\Gamma_1^\dagger)\tilde{H}\tilde{H}(I - \Gamma_1\Gamma_1^\dagger) \quad (35)$$

where  $Q_2$  is given by eq. 24 and  $H$  is an arbitrary matrix.

In the next section, we will discuss how the decentralized stochastic optimal control design posed in Section 2.2 can be formulated as a constrained optimal output feedback problem.

## 2.6 Equivalent Constrained Output Feedback Problem

Our primary objective is to develop methods for obtaining satisfactory decentralized control laws which are implementable and computable. To this end, we will pose the decentralized stochastic control problem under consideration as a constrained output feedback problem. By augmenting the state vector with the compensator states and restricting the output feedback gains to be block diagonal, the decentralized stochastic control problem defined by eqs. 1-18 can be posed as a constrained stochastic output feedback problem of the form [67].

$$x(k+1) = Ax(k) + Bu(k) + w(k) \quad (36)$$

$$y(k+1) = Cx(k+1) + v(k+1) \quad (37)$$

where the augmented state vector  $x(k)$  and the composite control  $u(k)$ , measurement  $y(k)$ , process noise  $w(k)$ , measurement noise  $v(k)$  vectors are defined by

$$x(k) = \begin{bmatrix} x_0(k) \\ z_1(k) \\ \vdots \\ z_p(k) \end{bmatrix} \quad w(k) = \begin{bmatrix} w_0(k) \\ w_1(k) \\ \vdots \\ w_p(k) \end{bmatrix} \quad (38)$$

where  $w_i(k)$ ,  $1 \leq i \leq p$ , are zero mean white noise sequences viewed as design parameters to account for implementation or other uncertainties.

$$u(k) = \begin{bmatrix} \tilde{u}_1(k) \\ u_1(k) \\ \vdots \\ \vdots \\ \tilde{u}_p(k) \\ u_p(k) \end{bmatrix} \quad y(k) = \begin{bmatrix} z_1(k) \\ y_1(k) \\ \vdots \\ \vdots \\ z_p(k) \\ y_p(k) \end{bmatrix} \quad v(k) = \begin{bmatrix} n_1(k) \\ v_1(k) \\ \vdots \\ \vdots \\ n_p(k) \\ v_p(k) \end{bmatrix} \quad (39)$$

where  $\tilde{u}_i(k)$ ,  $1 \leq i \leq p$ , are additional control vectors which will be optimized to design the finite order dynamic compensators. Note that the digital dynamic compensator state,  $z_i(k)$ , which will be computed by the subcontroller computer is known, and is included in the subcontroller measurement vector. The measurement noise,  $n_i(k)$ , associated with  $z_i(k)$ , may be selected as zero, if desired. Otherwise, round-off or other noise effects can be modelled. The variances for the composite process noise,  $w(k)$ , and the measurement noise  $v(k)$  will be denoted by  $W$  and  $V$ , respectively.

The composite system matrices are given by

$$A = \begin{bmatrix} A_e 0 & \dots & 0 \\ 0 & \ddots & \\ \vdots & & \\ 0 & & 0 \end{bmatrix} \quad (40)$$

$$B = [\bar{B}_1 \ \bar{B}_2 \ \dots \ \bar{B}_p] \text{ with} \quad (41)$$

$$\bar{B}_i = \begin{bmatrix} 0 & B_i \\ D_i & 0 \end{bmatrix} \quad D_i = [0 \dots 1_i \dots 0] \quad (42)$$

where  $1_i$  is the  $q_i \times q_i$  identity matrix.

$$C = \begin{bmatrix} \bar{C}_1 \\ \bar{C}_2 \\ \vdots \\ \bar{C}_p \end{bmatrix} \quad \text{with} \quad \bar{C}_i = \begin{bmatrix} 0 & D_i \\ C_i & 0 \end{bmatrix} \quad (43)$$

The augmented control vector,  $u(k)$ , can now be expressed as

$$u(k) = -K y(k) \quad (44)$$

where the gain  $K$  is constrained to be of block diagonal form; i.e.,

$$K = \begin{bmatrix} K_1 & 0 & \dots & 0 \\ 0 & K_2 & & \\ \vdots & & \ddots & \\ 0 & & & K_p \end{bmatrix} = \text{block diag } (K_i, 1 \leq i \leq p). \quad (45)$$

with

$$K_i = \begin{bmatrix} E_i & H_i \\ G_i & F_i \end{bmatrix} \quad , \quad 1 \leq i \leq p \quad (46)$$

Equations 36-46 imbed the implementable decentralized control problem into a constrained stochastic output feedback problem, the constraint being described in eq. 44.

The overall cost or objective function can be expressed in terms of the augmented system parameters in the form

$$J(K) = \lim_{N \rightarrow \infty} J_N(K) \quad (47)$$

$$J_N(K) = \frac{1}{2(N+1)} E \sum_{k=0}^N x'(k+1) Q x(k+1) + u'(k) R u(k) \quad (48)$$

$$Q = \sum_{i=1}^p s_i \bar{Q}_i \quad (49)$$

$$R = \text{block diag } (\bar{R}_i, 1 \leq i \leq p) \quad (50)$$

$$\bar{R}_i = \begin{bmatrix} 0 & 0 \\ 0 & R_i \end{bmatrix}, \quad \bar{Q}_i = \begin{bmatrix} Q_i & 0 \\ 0 & 0 \end{bmatrix} \quad 1 \leq i \leq p. \quad (51)$$

It is important to note that the problem will be solved for arbitrary  $Q$  and  $R$  of non-negative definite form, so that the decentralized control problem is a special case. That is, when desirable for design objectives, the compensator states and controls can be penalized by selecting  $Q_i$  and  $R_i$  appropriately.

As we are interested in the steady-state or infinite-time optimization problem, consider the limit in eq. 47. Let  $\tilde{D}$  be the set of gains (not necessarily block-diagonal) for which  $J_N(K)$  converges to a finite limit  $J(K)$ , and let  $\tilde{D}_b$  be the subset of  $\tilde{D}$  consisting

of block diagonal gains; i.e.,

$$\tilde{D} = \{K \mid \lim_{N \rightarrow \infty} J_N(K) < \infty\} \quad (52)$$

$$D_B = \{K \mid K \text{ in } \tilde{D} \text{ and } K \text{ is block diag.}\} \quad (53)$$

Similarly, we define the stabilizing gain set  $\tilde{S}$  and its subset  $\tilde{S}_B$  consisting of block diagonal stabilizing gains by:

$$\tilde{S} = \{K \mid \rho(A - BK) < 1\} \quad (54)$$

$$\tilde{S}_B = \{K \mid K \text{ in } \tilde{S} \text{ and } K \text{ is block diag.}\} \quad (55)$$

where  $\rho$  denotes the spectral radius.

We pose the optimal decentralized control problem as: Find a  $K^*$  in  $\tilde{S}_B$  which minimizes the cost function  $J(K)$  over  $\tilde{S}_B$ ; i.e.,

$$J(K^*) \leq J(K) \text{ for all } K \text{ in } \tilde{S}_B \quad (56)$$

In this formulation, when an optimal solution exists, it has a decentralized structure, it is implementable, and it stabilizes the closed-loop system.

## 2.7 Existence of an Optimal Decentralized Control

As mentioned previously, the general optimal decentralized LQG problem over the class of linear and nonlinear controllers can result in a nonlinear controller, and the existence of an optimal controller is not guaranteed. Thus, it is of interest to investigate conditions which ensure that an optimal controller exists for the decentralized control problem formulated here, before attempting to find an optimal controller which may not even exist. As will be shown in the following, for a large class of realistic problems, an optimal solution does indeed exist.

The major contribution of this section is the following (referring back to the previous section for notation) existence result:

- Block output stabilizability of  $(C, A, B)$ , positive definiteness of  $B'QB+R$  and  $CWC'+V$ , and the existence of a positive scalar  $s$  with positive semi-definite  $Q-sC'C$  and  $W-sBB'$  are a sufficient set of conditions for the existence of an optimal decentralized control.

The development of existence conditions and algorithms to obtain gains for the optimal implementable decentralized control problem closely follows the development for the unconstrained stochastic output feedback problem reported in [66]. We will make extensive use of these results by modifying them to account for the block diagonal gain constraint. The following results from the unconstrained output feedback problem are given for completeness. The proofs can be found in [66].

Lemma 4:  $\tilde{S}$  is a subset of  $\tilde{D}$  and for any  $K$  in  $\tilde{S}$ , the cost  $J(K)$  can be expressed as

$$J(K) = \frac{1}{2} \text{tr}\{P(K) W\} + \frac{1}{2} \text{tr}\{K' \bar{P}(K) K V\} \quad (57)$$

where

$$\tilde{P}(K) = B' P(K) B + R \quad (58)$$

and where  $P(K)$  is the solution of

$$P(K) = A(K)' P(K) A(K) + C' K' R K C + Q \quad (59)$$

with the closed-loop system matrix  $A(K)$  defined by

$$A(K) = A - B K C \quad (60)$$

Lemma 5:  $J(K)$  and  $P(K)$  are continuous on  $\tilde{S}$ .

Lemma 6: Let  $(C, A, B)$  be output stabilizable, and  $Q \geq s C' C$ ,  $W \geq s B B'$  for some  $s > 0$ . Then  $\tilde{S}(a) = \{K \text{ in } \tilde{S} | J(K) \leq a\}$  is closed for all  $a$  in  $R$ .

Lemma 7: Let  $B' Q B + R > 0$  and  $C' W C + V > 0$ . Then  $\tilde{S}(a)$  is bounded.

Using these results, we shall now show that an optimal decentralized control exists for a large class of problems. The following corollaries follow from the preceding lemmas.

Corollary 1:  $\tilde{S}_B$  is a subset of  $\tilde{D}_B$ .

Proof. By Lemma 4,  $\tilde{S}_B$  is a subset of  $\tilde{S}$ . It follows that  $J_N(K) \rightarrow J(K) < \infty$  and  $K$  is in  $\tilde{D}_B$ , whenever  $K$  is in  $\tilde{S}_B$ .

Corollary 2:  $J(K)$  and  $P(K)$  are continuous on  $\tilde{S}_B$ .

Proof: Since  $\tilde{S}_B$  is a subset of  $\tilde{S}$ , Lemma 5 holds for  $\tilde{S}_B$ .

Definition 2: The triplet  $(C, A, B)$  is block output stabilizable if

$$\rho(A - \sum_{i=1}^p B_i K_i C_i) < 1 \text{ for some block gain } K = \{K_1, K_2, \dots, K_p\}.$$

where  $\rho$  is the spectral radius.

Note that if  $K$  is block diagonal

$$A(K) = A - B K C = A - \sum_{i=1}^p B_i K_i C_i. \quad (61)$$

We will need the following definition for the fixed modes of a decentralized structure and the relationship between fixed modes and block output stabilizability.

Definition 3: Given a triplet  $(C, A, B)$  and the given set  $\tilde{K}$  of block diagonal matrices, the set of fixed modes of  $(C, A, B)$  with respect to  $\tilde{K}$  is defined

$$\Lambda = \bigcap_{K \in \tilde{K}} \sigma(A+BKC) \quad (62)$$

where  $\sigma(A)$  is the spectrum of  $A$ .

Lemma 6: Consider a decentralized system defined by the triplet  $(C, A, B)$  and the associated set  $\tilde{K}$  of block diagonal matrices. A necessary and sufficient condition for the block diagonal stabilizability of  $(C, A, B)$  is that the fixed modes of  $(C, A, B)$  be stable.

Proof: See Theorem 1 in [28].

Lemma 9: If  $(C, A, B)$  is block output stabilizable, then  $(C, A, B)$  is output stabilizable.

Proof: If  $(C, A, B)$  is block output stabilizable, then  $(C, A, B)$  can be stabilized by a block diagonal gain. Hence,  $(C, A, B)$  is output stabilizable.

Corollary 3: Let  $(C, A, B)$  be block output stabilizable, and  $Q \geq s C^* C$ ,  $W \geq s B B^*$  for some  $s > 0$ . Then  $\tilde{S}_B(a) = \{K \in \tilde{S}_B \mid J(K) \leq a\}$  is closed for all  $a$  in  $\mathbb{R}$ .

Proof: By Lemma 9,  $(C, A, B)$  is output stabilizable, and by Lemma 6,  $\tilde{S}(a)$  is closed. Note that  $\tilde{S}_B(a) = \tilde{S}(a) \cap \tilde{S}_B$ . Let  $K^i$  be in  $\tilde{S}_B(a)$  and  $K^i \rightarrow K$ . Since  $K^i$  is in  $\tilde{S}(a)$  also, and  $\tilde{S}(a)$  is closed, the limit  $K$  is in  $\tilde{S}(a)$ . As the limit of a block diagonal sequence of matrices is also block diagonal,  $K$  is in  $\tilde{S}_B$ . Hence,  $K$  is in  $\tilde{S}_B(a)$ , and  $\tilde{S}_B(a)$  is closed.

Corollary 4: Let  $B^* Q B + R > 0$ , and  $C^* W C + V > 0$ . Then  $\tilde{S}_B(a)$  is bounded.

Proof:  $\tilde{S}_B(a)$  is a subset of  $\tilde{S}(a)$  which is bounded under the assumed conditions. Hence  $\tilde{S}_B(a)$  is bounded.

Theorem 4. Let the fixed modes of  $(C, A, B)$  be stable,  $Q \geq s C^* C$ ,  $W \geq s B B^*$  for some  $s > 0$ ,  $B^* Q B + R > 0$ ,  $C^* W C + V > 0$ . Then there exists a  $K^*$  in  $\tilde{S}_B$  such that

$$J(K^*) \leq J(K), \text{ for all } K \text{ in } \tilde{S}_B$$

Proof. By Lemma 9,  $(C, A, B)$  is block output stabilizable. By Corollary 1,  $\tilde{S}_B$ , hence

$\tilde{D}_B$  is not empty. Thus, let  $J^* = \inf J(K) < \infty$ . Necessarily, there is a sequence  $\{K^i, i \geq 0\}$  in  $\tilde{S}_B$  such that  $J(K^i)^2 \downarrow J^*$ . Let  $\tilde{S}_{B0} = \{K \in \tilde{S}_B | J(K) \leq J(K^0)\}$ . By Corollaries 3 and 4,  $\tilde{S}_{B0}$  is closed and bounded. Using the Bolzano-Weierstrass theorem,  $\{K^i, i \geq 0\}$  has a limit point, say  $K^*$ , which belongs to  $\tilde{S}_{B0}$ . Hence,  $K^*$  is in  $\tilde{S}_B$ . By Corollary 2,  $J(K)$  is continuous on  $\tilde{S}_B$ ; therefore, if  $\{K^{ij}, j \geq 0\}$  is a subsequence converging to  $K^*$ , then  $J(K^{ij}) \downarrow J(K^*) = J^*$ .

Therefore, the infimum of  $J(K)$  over  $\tilde{S}_B$  is indeed attained at  $K^*$  in  $\tilde{S}_B$ , and  $K^*$  is an optimal solution to the decentralized control problem posed. In other words, under the conditions stated,  $J(K)$  has a minimum over  $\tilde{S}_B$ .

We note that the existence of a positive scalar  $s$  such that  $Q \geq s C'C$  and  $W \geq s BB'$  are precisely the same kind of weighting matrix selection procedures, ensuring low-sensitivity feedback controllers [68] and robust observers [69]. For instance, it is suggested in [69], that the system process noise should be selected such that

$$W = W_0 + q^2 BVB' \quad (63)$$

where  $W_0 \geq 0$ ,  $V > 0$ , and  $q$  is a non-zero scalar. The matrix  $W_0$  is the plant process noise covariance and the second term is the fictitious noise covariance to be added for robustness. Letting  $q^2=s$ ,  $V=I$ , and since  $W_0 \geq 0$ , we get  $W \geq s BB'$ . The other condition ( $Q \geq s C'C$ ) can be similarly obtained from [68]. Hence, these conditions are not at all constraining but rather good choices for a robust controller design, at least in the centralized case.

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<sup>2</sup>where  $\downarrow$  denotes monotonic convergence.

The existence conditions can be expressed in terms of the subsystem parameters for ease of interpretation.

Corollary 5: Let the fixed modes of  $(C, A, B)$  be stable, for some  $s > 0$ ,  $Q_i \geq s C_i^T C_i$ ,  $C_i$  and  $W \geq s B_i^T B_i$ ,  $1 \leq i \leq p$ ,  $B^T Q B + R > 0$ , and  $C^T W C + V > 0$ . Then there is a  $K^*$  in  $\tilde{S}_B$  such that

$$J(K^*) \leq J(K), \text{ for all } K \text{ in } \tilde{S}_B$$

Proof: Note that  $a_i > 0$ , so that  $\min a_i = a > 0$ .

$$Q = \sum_{i=1}^p a_i Q_i \geq s \sum_{i=1}^p a_i C_i^T C_i \geq s a \sum_{i=1}^p C_i^T C_i = s a C^T C, s a > 0.$$

Similarly, given the above hypothesis

$$W \geq s/p \sum_{i=1}^p B_i^T B_i = s/p B^T B \text{ with } s/p > 0$$

Thus, the conditions above imply the hypothesis of Theorem 4. So that  $J(K)$  has a minimum in  $\tilde{S}_B$ .

It should be noted that the conditions given in Theorem 4 and Corollary 5 are sufficient for the existence of an optimal decentralized control law, however, they are not necessary for the existence of an optimal solution. Also note that the question of uniqueness of the optimal gain is unresolved.

It can be seen that, for the decentralized control problem posed here, it is possible to find existence conditions which apply to a large class of optimization

problems. We note that the existence conditions include the case of no measurement noise ( $V=0$ ), provided that the process noise  $W$  is positive definite, the number of measurements are less than or equal to the number of states, and the measurement matrix  $C$  has full rank. Similarly, the case of no control penalty ( $R=0$ ) is also included if the state weighting matrix  $Q$  is positive definite, the number of controls are less than the number of states and the input transition matrix  $B$  has full rank. Moreover, the case of  $V=R=0$  simultaneously is also included, provided that the conditions above are both satisfied. In general, however, Theorem 4 places no restrictions on the number of measurements, states, and controls, and on the ranks of  $B$  and  $C$ . Thus, the case of multiple measurements of the same variable (i.e.  $C$  does not have full rank) as it would be expected in most decentralized control problems in which one measurement is available to more than one subsystem, will have an optimal stable solution if the existence conditions are met.

## 2.8 Incremental Cost and Necessary Conditions

The necessary conditions for the constrained output feedback problems have been previously explored [41]-[42]. These conditions have been obtained by casting the problem in a nonlinear programming framework. As the resulting necessary conditions are coupled nonlinear matrix equations, a reliable algorithm for obtaining their solution has not been available.

The approach taken in our study is based on the incremental cost, i.e., the total change in the cost due to a change in the gain. The expression for the incremental cost for the responding decentralized case is presented below. The incremental cost expression is used in obtaining the decentralized control law algorithms in the next

section. Our approach attempts to iteratively reduce the cost in order to locate a minimum, as opposed to solving the nonlinear matrix equations corresponding to the necessary conditions. These necessary conditions also follow immediately from the incremental cost. The expression for the incremental cost for the unconstrained output feedback problem is derived in [66] and repeated below for completeness.

Lemma 10: Let  $K$  and  $K + \Delta K$  be in  $\tilde{S}$ . Then the incremental cost is given by

$$\begin{aligned}\Delta J(K, \Delta K) = J(K + \Delta K) - J(K) &= \frac{1}{2} \text{tr}\{2 \Delta K' T(K, \Delta K)\} \\ &+ \frac{1}{2} \text{tr}\{\Delta K' \tilde{P}(K + \Delta K) \Delta K \tilde{S}(K)\}. \end{aligned} \quad (64)$$

$$T(K, \Delta K) = \tilde{P}(K + \Delta K) K \tilde{S}(K) - B' P(K + \Delta K) A S(K) C' \quad (65)$$

with

$$\tilde{S}(K) = C S(K) C' + V \quad (66)$$

where  $S(K)$  is the steady-state covariance of  $x(k)$  given by

$$S(K) = A(K) S(K) A(K)' + B K V K' B' + W. \quad (67)$$

The expression above holds on  $\tilde{S}$ , hence, it hold on every subset, and in particular on the set of block diagonal stabilizing gains  $\tilde{S}_b$ . Note from (64) that  $T(K, 0)$  is the gradient of  $J(K)$ .

The incremental cost expression for the decentralized case is presented next.

Corollary 6: Let  $K$  and  $K + \Delta K$  be in  $\tilde{S}_b$  with  $\Delta K$  block diagonal, i.e.,

$$\bar{D}(\bar{r}_i) \Delta K \bar{D}'(\bar{m}_j) = 0, \quad i \neq j, 1 \leq i, j \leq p \quad (68)$$

$$\bar{r}_i = r_i + q_i, \quad \bar{m}_i = m_i + q_i \quad (69)$$

and

$$D(\bar{r}_i) = [0 \dots I(\bar{r}_i) \dots 0] \quad (70)$$

where  $I(\bar{r}_i)$  is the  $\bar{r}_i \times \bar{r}_i$  identity matrix. Then the incremental cost is given by

$$\begin{aligned} \Delta J(K, \Delta K) &= \frac{1}{2} \sum_{i=1}^p \text{tr}\{2 \Delta K^* T_{ii}(K, \Delta K) \\ &\quad + \sum_{j=1}^p \Delta K^* \bar{P}_{ij}(K + \Delta K) \Delta K_j \bar{S}_{ji}(K)\}. \end{aligned} \quad (71)$$

$$T_{ii}(K, \Delta K) = D(\bar{r}_i) T(K, \Delta K) D'(\bar{m}_i). \quad (72)$$

$$\bar{P}_{ij}(K, \Delta K) = D(\bar{r}_i) \bar{P}(K + \Delta K) D'(\bar{r}_j). \quad (73)$$

$$\bar{S}_{ij}(K) = D(\bar{m}_i) \bar{S}(K) D'(\bar{m}_j) \quad (74)$$

Proof. Substituting the representation  $\Delta K = \sum_{i=1}^p D(\bar{r}_i) \Delta K_i D'(\bar{m}_i)$  for  $\Delta K$  in (64) and using trace identities, we get (71).

Note that the incremental cost above is the exact change, not the first order variation. The only restriction on  $K$  and  $K + \Delta K$  is that they meet the constraints and stabilize the system. From the definition of variation, we have

$$\mathcal{W}(K, \Delta K) = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} [J(K + \epsilon \Delta K) - J(K)] \quad (75)$$

Substituting the expression for the incremental cost into the definition above and using the continuity properties of  $J(K)$  given in Corollary 2, we can easily obtain the

gradient and hence the necessary conditions which are stated next.

Lemma 11:  $J(K)$  is continuously differentiable on the set of block diagonal stabilizing gains  $\tilde{S}_B$  and the gradient is given by

$$\frac{\partial J}{\partial K_i}(K) = T_{ii}(K, 0) \quad 1 \leq i \leq p \text{ and } K \text{ in } \tilde{S}_B. \quad (76)$$

The necessary conditions for optimality are

$$T_{ii}(K^*, 0) = 0 \quad 1 \leq i \leq p \text{ and } K^* \text{ in } \tilde{S}_B. \quad (77)$$

In the next section, we will present two algorithms for finding a minimal solution based on the optimal output feedback algorithm solving the cost functional directly through the use of the incremental cost as opposed to solving the necessary conditions.

## 2.9 Constrained Output Feedback Algorithms

As shown in the previous section, the necessary conditions for the posed constrained optimal output feedback problem are a coupled set of nonlinear matrix equations. The solution of these equations usually involves lengthy gradient search techniques in which the computation of large order Hessian matrices are required. Here, we will present two algorithms for solving the constrained output feedback problem based on the incremental cost.

The first constrained algorithm called the "subsystem iteration algorithm" here is a straightforward iterative application of the unconstrained output feedback algorithm [66] to each controller while closing the loop on the other controllers. We have

chosen this subsystem iteration algorithm as a benchmark. The reasons for considering this unconstrained output feedback algorithm are as follows: 1) First, its convergence for the unconstrained case has been proven analytically under realistically weak existence conditions; 2) Second, its speed of convergence is considerably better than others since it does not involve lengthy line searches.

The major result of this section is the second constrained algorithm, called the "decentralized control algorithm" here, which is a generalization of the unconstrained one to constrained systems in which the direction suggested by the constrained incremental cost given by eqs. (73)-(75) is used. The convergence of this algorithm is proved in Section 2.10. We first present the subsystem iteration algorithm.

### Subsystem Iteration Algorithm

- I. Choose a block diagonal  $K_o = \text{block diag } \{[K_{oi}], i=1, \dots, p\}$  such that  $A(K_o)$  is stable and set  $n=1$ .
- II. Choose an index  $j$  from  $\{1, \dots, p\}$ . Solve the unconstrained output feedback problem for the system  $(A - \sum_{i \neq j} B_i K_{oi} C_i, B_j, C_j)$  with weighting matrices  $Q + \sum_{i \neq j} C_i' K_{oi} R_i K_{oi} C_i$  and  $R_j$ , process noise  $W + \sum_{i \neq j} B_i K_{oi} V_i K_{oi} B_i'$  and measurement noise  $V_j$  using the algorithm for the unconstrained optimal output problem described in [65] with an initial guess  $K_{oj}$  to obtain  $K_{nj}$  set  $K_{oj} = K_{nj}$ .
- III. Repeat Step II for all of the other indices in  $\{1, \dots, p\}$  to obtain  $\{[K_{ni}], i=1, \dots, p\}$ .
- IV. Evaluate the incremental cost  $J(K_n) - J(K_{n-1})$  given by eq. 76 and the gradient  $\frac{\partial J}{\partial K_{ni}}(K_n)$  for  $i=1, \dots, p$  given by Lemma 11. Test for convergence of the incremental cost and gradient. Stop if selected convergence criteria are satisfied; otherwise, set  $n=n+1$  and go to Step II.

As implied by Theorem 4, when the number of controls and measurements are both less than the number of states,  $B$  and  $C$  have full rank, and  $Q$  and  $W$  are positive definite, then block output stabilizability of  $(C, A, B)$  is a sufficient condition for the existence of a minimal stabilizing gain in Step II of the algorithm above. Under these and the more general conditions stated in Theorem 4, it is easy to see that  $\{J(K_n)\}$  is a monotonically decreasing sequence. Since the sequence is also bounded from below, the costs will converge to a minimum. Since the set of block diagonal stabilizing gains over which the optimization is performed can be an open set, it needs further to be shown that the set of block diagonal stabilizing gains with a bounded cost is bounded so that the minimum block diagonal gain is attained at an interior point.

The second algorithm is obtained by exploiting the "almost quadratic" form of the incremental cost function for the constrained output feedback problem given by eq. (76). The algorithm is started with a stabilizing block diagonal gain  $K_0$  and iteratively produces a block diagonal gain sequence  $\{K_n, n=1, 2, \dots\}$  in which the direction  $d(K_n) = K_{n+1} - K_n$  used at each iteration is the one suggested by the constrained incremental cost expression.

Before presenting the algorithm, however, we will give the following results which will insure the invertibility of a certain composite matrix,  $U(K)$ , at one step of the algorithm. We also note that the conditions guaranteeing the invertibility of this matrix are precisely the same ones dictated by the existence conditions in Theorem 4.

Lemma 12: Consider the positive semi-definite matrices  $\bar{P}(K)$  and  $\bar{S}(K)$ , defined in eqs. (63) and (71), respectively. If  $B'QB + R > 0$  and  $CWC' + V > 0$ , then  $\bar{P}(K) > 0$  and  $\bar{S}(K) > 0$  for any gain matrix  $K$ .

Proof: From eq. 64, we have  $P(K) \geq Q$  which implies  $B'P(K)B \geq B'QB$ . So  $B'P(K)B + R \geq B'QB + R$ . Hence,  $\bar{P}(K) \geq B'QB + R > 0$  which implies  $\bar{P}(K) > 0$ . Similarly, from eq. 72, we have  $S(K) \geq W$  which implies  $CS(K)C' + V = \bar{S}(K) \geq CWC' + V > 0$ .

Lemma 13: Consider a positive-definite matrix  $S$ . If  $S_{ii}$  is an arbitrary block diagonal partitioning of  $S$ , (i.e.  $S_{ii} = D_i S D_i'$ , where  $D_i$  is defined by eq. 42), then  $S_{ii} > 0$  for all  $i$ .

Proof. Since  $x'Sx > 0$  for all  $x$ , we have  $x'D_i S D_i' x > 0$  which implies  $S_{ii} > 0$ .

Lemma 14: Consider the symmetric composite matrix  $U(K)$  formed from the partitions defined by

$$U_{qr}(K) = \bar{P}_{qr}(K) \otimes \bar{S}_{qr}(K) \quad q,r = 1,2,\dots,p \quad (78)$$

where  $\otimes$  is the direct product and  $\bar{P}(K)$  and  $\bar{S}(K)$  are defined by eqs. 63 and 71. If,  $B'QB + R > 0$  and  $CWC' + V > 0$ , then  $U(K)$  is positive-definite for any matrix  $K$ .

Proof: From Lemma 12, we have  $\bar{P}(K) > 0$  and  $\bar{S}(K) > 0$ . Lemma 13 implies that  $\bar{S}_{ii}(K) > 0$  for  $i=1,2,\dots,p$ . By Lemma on pg. 870 in [70] and by Lemma 1 in [21], we have  $U(K) > 0$ .

We now present the decentralized control algorithm which is the generalization of the unconstrained output feedback algorithm to the constrained case.

#### Decentralized Control Algorithm

I. Choose a block diagonal  $K_o$  such that  $A(K_o)$  is stable,  $\alpha_o$  in  $[0,1]$ ,  $z>1$ , and set  $n=0$ .

II. Solve the following for  $S(K_n)$

$$S(K_n) = A(K_n)S(K_n)A'(K_n) + BK_nVK_n'B' + W$$

III. Solve the following for  $P(K_n)$

$$P(K_n) = A'(K_n)P(K_n)A(K_n) + C'K_n'RK_nC + Q$$

IV. Compute  $\bar{P}(K_n)$ ,  $\bar{S}(K_n)$

$$\bar{P}(K_n) = B'P(K_n)B + R$$

$$\bar{S}(K_n) = CS(K_n)C' + V$$

V. Form the symmetric composite matrix  $U(K_n)$  defined by eq. (78) from the partitions

$$U_{qr}(K_n) = \bar{P}_{qr}(K_n) \otimes \bar{S}_{qr}(K_n) \quad q,r=1,\dots,p$$

where  $\otimes$  is the matrix direct product. If  $U_n$  is not positive-definite, go to Step 9.

VI. Form a column vector  $s(K_n)$  by storing the row vectors of  $B'P(K_n)A(K_n)S(K_n)C'$  sequentially and denote a column gain vector  $k_{new}$  obtained similarly from the rows of  $K_{new}$ . Compute  $k_{new}$  and  $d(K_n)$  from

$$U(K_n) k_{new} = s(K_n)$$

$$d(K_n) = K_{new} - K_n$$

VII. Compute  $K_{n+1}$

$$K_{n+1} = K_n + \alpha_n d(K_n)$$

VIII. Evaluate cost function

$$J(K_n) = 1/2 \operatorname{tr}[P(K_n)W + K_n' \bar{P}(K_n)K_n V]$$

If  $n=0$  set  $n=1$ ,  $\alpha_{n+1} = \alpha_n$ , go to Step II.

If  $J(K_n)$  is negative or either of  $S(K_n)$  or  $P(K_n)$  is not positive semidefinite, go to Step IX. If  $J(K_n) - J(K_{n-1})$  is negative, go to Step X, otherwise go to Step IX.

IX. Set  $\alpha_n = \alpha_n/z$ ,  $K_n = K_{n-1}$ ,  $d(K_n) = d(K_{n-1})$ .

Compute  $K_{n+1} = K_n + \alpha_n d(K_n)$ . Set  $\alpha_{n+1} = \alpha_n$ ,  $n=n+1$ , and go to step II.

X. Compute the constrained gradient

$$\frac{\partial J}{\partial K_{ni}}(K_n) = D_i T(K_n, 0) D_i' \quad i=1,\dots,p$$

If norm of  $\frac{\partial J}{\partial K_{ni}}$  and  $J(K_n) - J(K_{n-1})$  are less than some convergence

criterion stop, otherwise, go to Step II.

In the decentralized control algorithm above, the objective is to choose a large positive initial  $\alpha$  which makes the algorithm stable. Steps V and VIII check the lack of convergence and the necessary reduction in  $\alpha$  is made in Step IX. The equations in Step II and III can be solved using the Bartels-Stewart algorithm available in control software packages such as ORACLS [72].

In the next section, we will give our results concerning the convergence of the decentralized control algorithm. In the next chapter, we will present simulation results for a nontrivial example problem, in which both algorithms are used to synthesize a coordinated design of a flight and engine control subsystem.

## 2.10 Decentralized Convergence Results

In this section, we show that under the existence conditions outlined in Section 2.6, the decentralized control algorithm given in the previous section converges at least to a local minimum. Theorem 5 is the major result of this section. Basically, it is shown that the decentralized control algorithm produces a block diagonal stabilizing gain at each iteration and the sequence of gains converge to a block diagonal stabilizing gain, satisfying the necessary condition given in Section 2.8. Furthermore, it is proved that the cost sequence corresponding to the gain sequence converges monotonically. Any stabilizing block diagonal gain can be used to start the algorithm. Theorem 5 guarantees that a block diagonal gain can be found, such that, the associated gradient can be made as small as possible.

Theorem 5 assures that the new gain at each iteration of the algorithm will not fall outside the stability region. Furthermore, only a single constant value of the parameter  $\alpha$  is sufficient to obtain the convergent gain sequence. Therefore, it is not necessary to conduct lengthy line searches at each iteration. Although the selection of  $\alpha$  may require a number of decreases in this parameter, Theorem 5 assures that a value of  $\alpha$ , for which the algorithm is stable, will be reached in a finite number of steps.

Lemma 15: Let the existence conditions in Theorem 4 hold. Then the set  $\tilde{S}_0$  defined by

$$\tilde{S}_0 = \{K \text{ in } \tilde{S}_B \mid 0 \leq J(K) \leq J(K_0)\} \quad (79)$$

is closed and bounded where  $K_0$  is an arbitrary element of  $\tilde{S}_B$ .

Proof: By Corollary 3,  $\tilde{S}_0$  is closed. By Corollary 4, it is also bounded.

Theorem 5: Let the existence conditions in Theorem 4 hold. Then there exists a scalar  $b$  in  $(0,1)$  and  $K^*$  in  $\tilde{S}_B$  such that

$$J(K_n) \downarrow J(K^*) \quad (80)$$

and

$$\lim_{n \rightarrow \infty} \frac{\partial J}{\partial K_i} (K_n) = \frac{\partial J}{\partial K_i} (K^*) \text{ for } i=1,2,\dots,p \quad (81)$$

where the block diagonal gain sequence  $K_n = \text{block diag } \{[K_{ni}], i=1,2,\dots,p\}$  is defined by

$$K_{(n+1)i} = K_{ni} + \alpha d(K_{ni}) \quad i=1,2,\dots,p \quad (82)$$

for  $\alpha$  in  $(0, b)$  and arbitrary  $K_0$  in  $\tilde{S}_b$ . The sequence  $\{[d(K_{n_i})]\}$ ,  $i=1, 2, \dots, p\}$  is the solution of

$$T_{ii}(K_n) + \sum_{j=1}^p \bar{P}_{ij}(K_n) d(K_{n_j}) \bar{S}_{ji}(K_n) = 0 \quad (83)$$

and  $K^*$  satisfies the necessary conditions of optimality.

Proof: We will first show that the set

$$\tilde{S}_{\alpha} = \{K + \alpha d(K) \mid K \text{ in } \tilde{S}_0 \text{ and } \alpha \text{ in } [0, a]\} \quad (84)$$

is a subset of  $\tilde{S}_b$  for same  $a > 0$ . Suppose that there is no such  $a > 0$ . Then if we can construct a sequence  $a_i \downarrow 0$  and a sequence  $\{K_i\}$  in  $\tilde{S}_0$  such that

$$\rho A(K_i + a_i d(K_i)) \geq 1 \quad (85)$$

Since  $\tilde{S}_0$  is closed and bounded by Lemma 15  $\{K_i\}$  has a limit point  $\bar{K}$  in  $\tilde{S}_0$ . Note that under the assumed existence conditions, the matrices  $\bar{P}(K)$  and  $\bar{S}(K)$  for  $K$  in  $\tilde{S}_b$  are positive definite since

$$\bar{P}(K) \geq B' B + R > 0 \quad (86)$$

$$\bar{S}(K) \geq C' C + V > 0 \quad (87)$$

Now forming composite column vectors,  $t(K)$  and  $\bar{d}(K)$ , from the rows of  $T_{ii}(K)$  and  $d(K_i)$ ,  $i=1, 2, \dots, p$ , eq. (83) can be written as a vector equation of the form

$$t(K) + U(K) \bar{d}(K) = 0 \quad (88)$$

where  $U(K)$  is defined by eq. (78).

By Lemma 14,  $U(K)$  is positive definite, so that  $d(K)$  exists and is continuous on  $\tilde{S}_B$ . Hence,  $d(K)$  is continuous on the closed and bounded set  $\tilde{S}_o$ . Since  $\rho(A(K))$  is also continuous, for some subsequence

$$K_{ij} + a_{ij} d(K_{ij}) \rightarrow \bar{K} \quad (89)$$

$$\rho(A(K_{ij}) + a_{ij} d(K_{ij})) \rightarrow \rho(\bar{A}(\bar{K})) \geq 1$$

which is a contradiction since  $\bar{K}$  is in  $\tilde{S}_o$ . Therefore,  $\tilde{S}_{oo}$  is a subset of  $\tilde{S}_B$  for some  $a > 0$  which will be considered fixed.

Since  $\tilde{S}_{oo}$  is the inverse image of a closed set, and since  $d(K)$  is continuous, it follows then  $\tilde{S}_{oo}$  is closed and bounded. Since  $P(K)$  is continuously differentiable over  $\tilde{S}_{oo}$ , it can be shown that for some finite  $M$

$$\|\Delta P(K, \alpha d(K))\| \leq \alpha M \|d(K)\| \quad (90)$$

where  $K$  is in  $\tilde{S}_o$  and  $\alpha$  is in  $[0, a]$ .

Now, using the following decomposition

$$T_{ii}(K, \Delta K) = T_{ii}(K, 0) + \Delta T_{ii}(K, \Delta K) \quad (91)$$

where  $T_{ii}(K, 0)$  is defined by eqs. (69), (78), i.e.

$$T_{ii}(K, 0) = D(\bar{r}_i) \{ \bar{P}(K) \bar{K} \bar{S}(K) - \bar{B}' \bar{P}(K) \bar{A} \bar{S}(K) \bar{C}' \} D'(\bar{m}_i) \quad (92)$$

$$\Delta T_{ii}(K, \Delta K) = D(\bar{r}_i) \{ \bar{B}' \Delta P(K, \Delta K) \bar{B} \bar{K} \bar{S}(K) - \bar{B}' \Delta P(K, \Delta K) \bar{A} \bar{S}(K) \bar{C}' \} D'(\bar{m}_i) \quad (93)$$

and substituting eq. (91) into eq. (75), letting  $\Delta K_i = \alpha d(K_i)$ , and rearranging terms using eq. (83), we get.

$$\Delta J(K, \alpha d(K)) = 1/2[-\alpha(2-\alpha) Q(K) + \alpha^2 W(K, \alpha)] \quad (94)$$

K in  $\tilde{S}_0$  and  $\alpha$  in  $\{0, a\}$

where

$$Q(K) = \text{tr}\{\sum_{i=1}^P d'(K_i) \sum_{j=1}^P \bar{P}_{ij}(K) d(K_j) \bar{S}_{ji}(K)\} \quad (95)$$

$$W(K) = \frac{2}{\alpha}$$

$$\text{tr}\{\sum_{i=1}^P d'(K_i) [\Delta T_{ii}(K, \alpha d(K)) + \sum_{j=1}^P \Delta P_j(K, \alpha d(K))]$$

Now, eq. (95) can be written as

$$Q(K) = \|P(K) d(K) S(K)\|_2 \quad (97)$$

where  $P(K)$  and  $S(K)$  are the square roots of  $\bar{P}(K)$  and  $\bar{S}(K)$ , respectively, and where  $\|\cdot\|_2$  is the euclidean matrix norm [63]. Since in finite dimensional vector spaces, any two norms are equivalent, it can be shown from eqs. (86),(87),(97) and the definition of norm for some  $M_2 > 0$ .

$$Q(K) \geq M_2 \|d(K)\|^2 \geq 0 \text{ for } K \text{ in } \tilde{S}_0 \quad (98)$$

where  $\|\cdot\|$  is an arbitrary matrix norm. On the other hand, using eq. (90 and 96),

$$|W(K, \alpha)| \leq M_1 \|d(K)\|^2 \text{ for } K \text{ in } \tilde{S}_0 \quad (99)$$

for some finite  $M_1$ . It follows that

$$|W(K, \alpha)| \leq M_3 Q(K) \quad K \text{ in } \tilde{S}_0 \quad (100)$$

where  $M_3 = M_1/M_2$ . Now select  $b$  in  $(0, 1]^3$  such that  $b \leq a$  and  $b \leq 1/M_3$ , and let  $\alpha$  be in

<sup>3</sup>where  $(0, 1]$  denotes the interval on the real line open at 0 and closed at 1.

(0,b]. Substituting eq. (100) into (94)

$$\Delta J(K, \alpha d(K)) \leq 1/2 [-\alpha(1-\alpha M_3)Q(K)] \quad (101)$$

for  $K$  in  $\tilde{S}_0$ . Since  $Q(K) > 0$  whenever  $d(K) \neq 0$  and  $0 < \alpha < 1/M_3$ , we have

$$\Delta J(K, \alpha d(K)) \leq 0 \text{ for } K \text{ in } \tilde{S}_0 \quad (102)$$

for  $\alpha$  in (0,b]. Equality holds if and only if  $d(K)=0$ .

It follows that if  $K$  is  $\tilde{S}_0$  and  $\alpha$  is in (0,b], then  $K+\alpha d(K)$  is in  $\tilde{S}_0$ . Therefore,  $\{K_i\}$  is a subset of  $\tilde{S}_0$ . Since  $J(K_i)$  is monotone and bounded from below, it converges to a limit point  $K^*$  in  $\tilde{S}_0$ , i.e.

$$J(K_i) \downarrow J(K^*) \quad (103)$$

From, eqs. (98) and (101), we have

$$0 \leq M_2 \|d(K_i)\|^2 \leq Q(K_i) \leq M_4 \Delta J(K_i, \alpha d(K_i)) \quad (104)$$

for some positive  $M_4$ . Since the cost increments must go to zero,  $Q(K_i)$  and  $d(K_i)$  must vanish in the limit. Since

$$Q(K_i) = \text{tr} \left\{ \sum_{i=1}^p 2\alpha d'(K_i) [T_{ii}(K, 0) + \sum_{j=1}^p \bar{P}_{ij}(K) \alpha d(K_j) \bar{S}_{jj}] \right\} \quad (105)$$

the gradient  $T_{ii}(K, 0)$  also converges to zero. Since the gradient is continuous, we have eq. (81) and the proof complete.

In the next chapter, we will present the simulation results for the two constrained output feedback algorithms.

### 3. APPLICATION TO AN AIRCRAFT INTEGRATED FLIGHT AND ENGINE CONTROL PROBLEM

In this section, we will present simulation results for an aircraft problem in which the two constrained output feedback algorithms of Section 2.9 are used to design integrated flight and engine control subsystems. The example system, taken from [73], represents the combined dynamics of an airframe and propulsion system for a typical twin-engine, advanced fighter aircraft.

#### 3.1 Simulation and Algorithm Comparison Results

The airframe dynamics are the linearized aircraft longitudinal equations of motion at a specified flight condition and are of the form:

$$\dot{x}_e(t) = A A_e x_e(t) + A A_e x_e(t) + B A_e u_e(t) + B A_e u_e(t) + w_e(t) \quad (106)$$

$$y_e(t) = x_e(t) + v_e(t) \quad (107)$$

where  $x_e$  is the five dimensional airframe state vector composed of the velocity  $v$ , angle of attack  $\alpha$ , pitch rate  $p$ , pitch attitude  $\Theta$ , and altitude  $h$  variables,  $u_e$  is the airframe control vector comprised of the horizontal stabilizer  $\delta_e$ ,  $y_e$  is the airframe measurement vector and  $y_e$  is engine measurement<sup>4</sup> vector. The engine dynamics are obtained through model reduction techniques and are of the form:

$$\dot{x}_e(t) = A E_e x_e(t) + A E_e x_e(t) + B E_e u_e(t) + w_e(t) \quad (108)$$

$$y_e(t) = x_e(t) + v_e(t) \quad (109)$$

where  $x_e$  is the five-dimensional engine state consisting of the fan speed  $N_1$ .

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<sup>4</sup>In order to treat the same problem in [73], actual airframe and engine measurements are not used. Of course, controllers using such measurements can be designed using the method developed here.

compressor speed N2, augmentor pressure P5, main burner fuel flow  $W_f$ , and compressor discharge pressure P2 variables.  $u_e$  is the engine control vector composed of the commanded fuel flow  $W_{fc}$ , nozzle area A, inlet guide valve CIVV, rear compressor variable valve RCVV, and compressor bleed variables BLC, and  $y_e$  is the engine measurement vector. System matrices above are listed in Appendix C in [73]. Linearization equilibrium point corresponds to a Mach number of 0.9, altitude of 13.72 km, and thrust of 12,833 newtons.

The objective of the flight control subsystem is to improve longitudinal performance through coordinated use of interacting aerodynamic and propulsive forces and moments. The performance index of the flight control subsystem is of the form:

$$J_e = \lim_{t_f} E \int_{t_0}^{t_f} z_e^T(t) Q_e z_e(t) + u_e^T(t) R_e u_e(t) dt \quad (110)$$

where  $z_e$  is the flight control response vector composed of velocity, glide path angle, and pitch rate variables, and  $u_e$  is the flight control vector composed of horizontal stabilizer and thrust variables. The response vector is a linear combination of the airframe state and control variables [73]. At this operating point, engine control subsystem objective is described by

$$J_e = \lim_{t_f} \frac{1}{t_f} E \int_0^{t_f} x_e^T(t) Q_e x_e(t) + u_e^T(t) R_e u_e(t) dt \quad (111)$$

where  $x_e$  and  $u_e$  are the engine state and control variables defined by eqs. (108)-(109). The weighting matrices  $Q_e$ ,  $R_e$ ,  $Q_e$ , and  $R_e$  are given in Appendix C in [73]. The airframe, engine and subsystem performance objectives have been fused into one single objective function as done in Section 2.6 by

$$J = c_1 J_e + c_2 J_e \quad (112)$$

The values for the scalars used are from [73]. The continuous cost functional has been discretized over a sampling interval of 0.1 sec as done in [61] and an equivalent sampled-data quadratic performance index has been found. The following constrained output feedback problem is then posed. Find  $K_o^*$  and  $K_e^*$  minimizing  $J$  over gains satisfying the constraint

$$u_o(k) = -K_o y_o(k) \quad (113)$$

$$u_e(k) = -K_e y_e(k) \quad (114)$$

The controller constraint above corresponds to the controller structure used in [73] as Strategy 2. We also note that the full state feedback controller optimizing the flight and engine control cost functionals separately (i.e., neglecting the interaction between the airframe and engine dynamics) has a very unsatisfactory performance for this problem (Strategy 1 in [73]). That is, the interaction between the subsystems in this example is significant and its neglect leads to performance degradation.

Figure 2 shows the performance of the closed-loop system employing the full-state continuous regulator control optimizing the cost functional 112. Eigenvalues for the closed-loop system employing these full-state feedback gains are given in Table 1.

The control gain values obtained for this optimal limited-state (eqs. 113-114) feedback problem using subsystem iteration are given in Table 2. The corresponding closed-loop system eigenvalues can be seen in Table 4. In Figure 3, the performance results are presented for the closed-loop system employing the optimal digital constrained control gains computed using the subsystem iteration algorithm. As shown in this figure, the flight control response ( $v, a, q, \theta, h$ ) of the optimal integrated digital

constrained feedback controller is remarkably close to that of the optimal continuous full-state regulator. It is also seen that the integrated design uses a slightly different engine control strategy than that of the full-state regulator. However, the response of the engine state variables ( $N1, N2, P5, Wf, P2$ ) for the integrated design is significantly better than the results obtained with the suboptimal control Strategy 2 in [73], which are given in Figure 4. As seen from the figure, the engine state variable response of the suboptimal strategy is considerably degraded compared to the optimal decentralized (Figure 3) and full-state regulator (Figure 2) cases. We also note that the optimal decentralized flight controller gains have lower values than those used in the suboptimal strategy.

The same problem has also been solved using the decentralized control law algorithm. Table 4 contains a comparison of the two algorithms. For the same stopping criterion, decentralized control algorithm was approximately three times faster than the subsystem iteration algorithm. As seen from Table 4, the decentralized algorithm converged after 51 iterations with an  $\alpha=0.3$ . On the other hand, subsystem iteration algorithm converged after 24 first subsystem (first order) and 127 second subsystem (fourth order) iterations with  $\alpha$  values of 0.8 and 0.2, respectively. It is interesting to note that a lower value of  $\alpha$  was required for the second subsystem iteration in this algorithm.

The convergence results are to be expected since the decentralized control algorithm uses the direction (better than that used by the other algorithm) suggested by the incremental cost. On the other hand, the order of the linear equation to be solved in the subsystem iteration algorithm will always be lower than that involved in

the decentralized control algorithm.

### 3.2 A Note on Nonlinear Decentralized Control

As demonstrated by Witsenhausen [37], for decentralized LQG problems, a nonlinear control can produce a smaller cost than the optimal linear control. As the formulation developed here restricts the optimization to linear implementable controllers, it is reasonable to ask whether a nonlinear controller can produce significantly better performance than the optimal linear controller obtained.

An approach to analyze this question is to obtain a lower bound on the optimal nonlinear decentralized cost. For the purpose of this discussion, we disregard the fact that a quadratic cost function does not always express all the design objectives. Thus, for LQG problems, a lower bound can be obtained by simply removing the constraint that the controller have a decentralized structure. In other words, the optimal full-state feedback LQG controller minimizes the cost over both linear and nonlinear controllers, including decentralized controllers, and is easily computable. This optimal controller therefore produces a cost at least as low as the optimal nonlinear decentralized controller, thus providing the desired lower bound.

In the example considered here for an integrated engine and flight path control, the performance of the implementable, linear, decentralized control and that of the full-state feedback LQG controller are almost the same. Both controllers simulated appear to provide almost the same control strategies and performance. The performance of the optimal nonlinear decentralized controller is bounded below by the full-state feedback controller and bounded above by the linear decentralized

controller. However, as the performance of the linear decentralized system is so close to the optimal LQG solution, the loss in performance appears to be minimal.

The approach to this analysis is quite general. The optimal full-state feedback LQG controller can be computed and simulated, and its performance compared to that of the optimal implementable, linear, decentralized controller obtained by the formulation presented in this study. This provides an indication on the extent to which a nonlinear law might improve performance.

Whereas, for highly nonlinear plants, a nonlinear controller is likely to produce better results. For linear plants with reasonable feedback vectors, we think that linear feedback is likely to produce acceptable performance without resorting to optimal nonlinear decentralized controllers for which efficient algorithms are not available. Thus, we think that for linear plants, appropriate selection of the feedback vector and the order of the dynamic compensation can often produce the desired performance.

#### 4. CONCLUSIONS AND RECOMMENDATIONS

The coordinated design of control subsystems has been investigated in the context of the optimal stochastic decentralized control problem for discrete systems. The problem is formulated so that the optimal controller is implementable and linear, has the desired decentralized structure, stabilizes the closed-loop system, and produces a coordinated system where the subcontrollers cooperate in the performance of their individual tasks to the extent possible.

The problem is posed as a constrained stochastic output feedback problem, where the gain is constrained to be block diagonal. In this formulation, the decentralized controller is constrained to be linear, with finite order, implementable dynamic compensators acting as subcontrollers. Subsystem controller performance objectives are optimized over the parameters of their fixed order linear dynamic structure. The notion of noninteraction for quadratic cost functionals are introduced, and necessary and sufficient conditions are given ensuring noninteraction between subsystem controller objectives.

This formulation produces two further benefits. The first is that the question of the existence of an optimal, implementable, decentralized control law can be satisfactorily treated. Sufficient conditions which guarantee the existence of an optimal controller are obtained. These conditions include a large class of optimization problems, and are expressed in terms of the known system and objective function parameters so that the existence of a solution can be easily determined at the outset of the design process. The question of uniqueness, however, remains to be resolved.

The second benefit is that the formulation leads to two constrained output feedback algorithms. The first of these algorithms called the subsystem iteration algorithm is an iterative application of an unconstrained output feedback algorithm. The second one called the decentralized control algorithm, is a generalization of this unconstrained algorithm to the decentralized case and is based on the constrained incremental cost direction. It is proved that the decentralized control algorithm converges to a solution under the sufficiency conditions ensuring the existence of an optimal solution.

The formulation is used to obtain an integrated design for an aircraft engine control and flight path control problem by using the software implementation of the two algorithms. The convergence rate of the decentralized control algorithm for this example is shown to be significantly faster than that of the subsystem iteration algorithm. A linear simulation shows excellent performance in comparison to the unconstrained full-state feedback LQG controller. This demonstration is used to analyze the significance and necessity of nonlinear decentralized controllers. These simulation results, along with the two uncooperating subsystems example, show the significant potential of the coordinated control approach in minimizing fuel expenditures.

The following is our list of recommendations for future study.

- o Although the conditions for the existence of an optimal solution have been found, the minimal restrictions on the system parameters assuring the uniqueness of the solution remain to be resolved. The importance of the uniqueness result is that the decentralized control law algorithm would converge to the global minimum if the solution is unique, since it has been established that the algorithm converges to at least a local minimum.

- o The cost functionals used in this study are average quadratic costs which, therefore, do not depend on the initial conditions. While the average cost is adequate for quite a number of control applications, it is desirable to extend the results to quadratic cost functionals which depend on initial conditions. This would be especially important for applications having stringent transient response requirements.
- o The developed constrained output feedback algorithms require an initial block stabilizing gain. Hence, it is desirable to find an easy method of computing an initial gain under the shown existence conditions.
- o It is also of interest to find the conditions on system parameters so that the constrained output feedback optimization yields a stable compensator.
- o Robustness properties of the developed decentralized control law algorithms need to be investigated.
- o Finally, after the issues above are resolved, we recommend that the decentralized control law algorithm be tested in a comprehensive integrated aircraft control design problem.

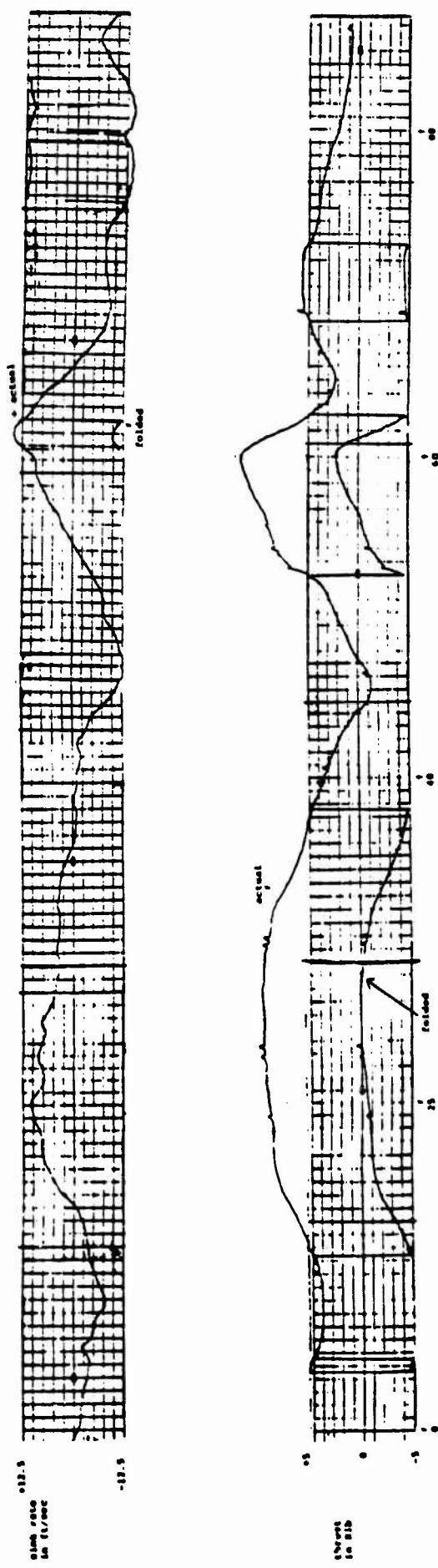


FIG. 1. SINK RATE AND THRUST TIME HISTORIES

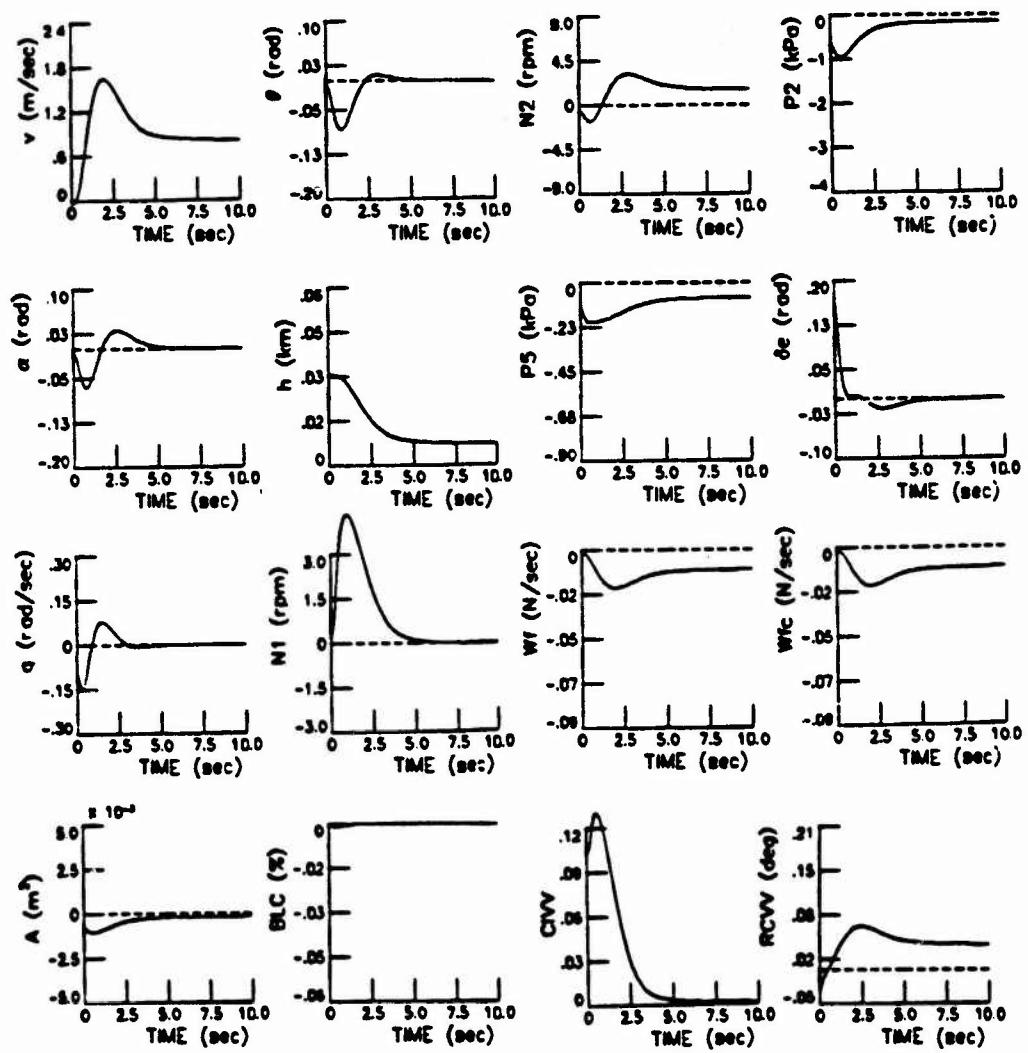


FIG. 2. OPTIMAL CONTINUOUS FULL-STATE REGULATOR PERFORMANCE

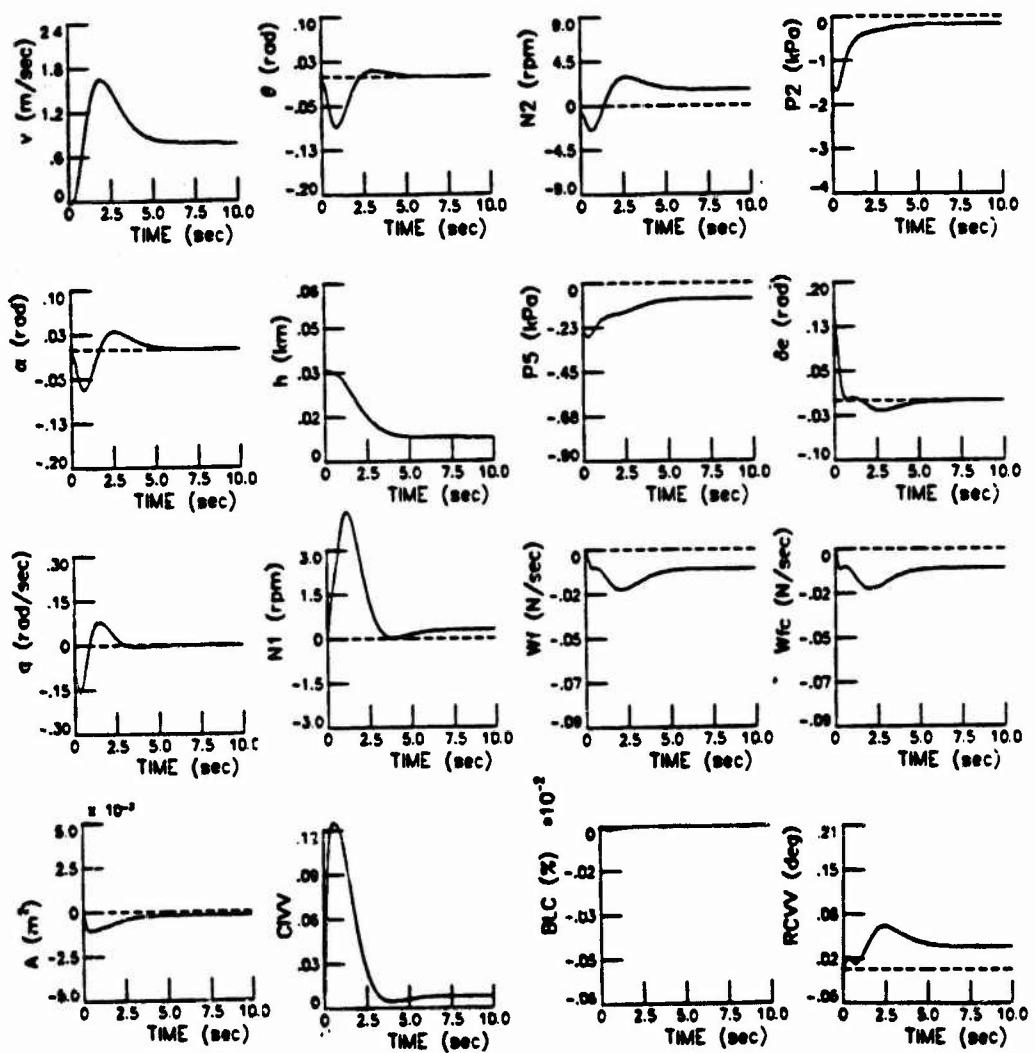


FIG. 3 OPTIMAL DISCRETE-TIME DECENTRALIZED CONTROLLER PERFORMANCE

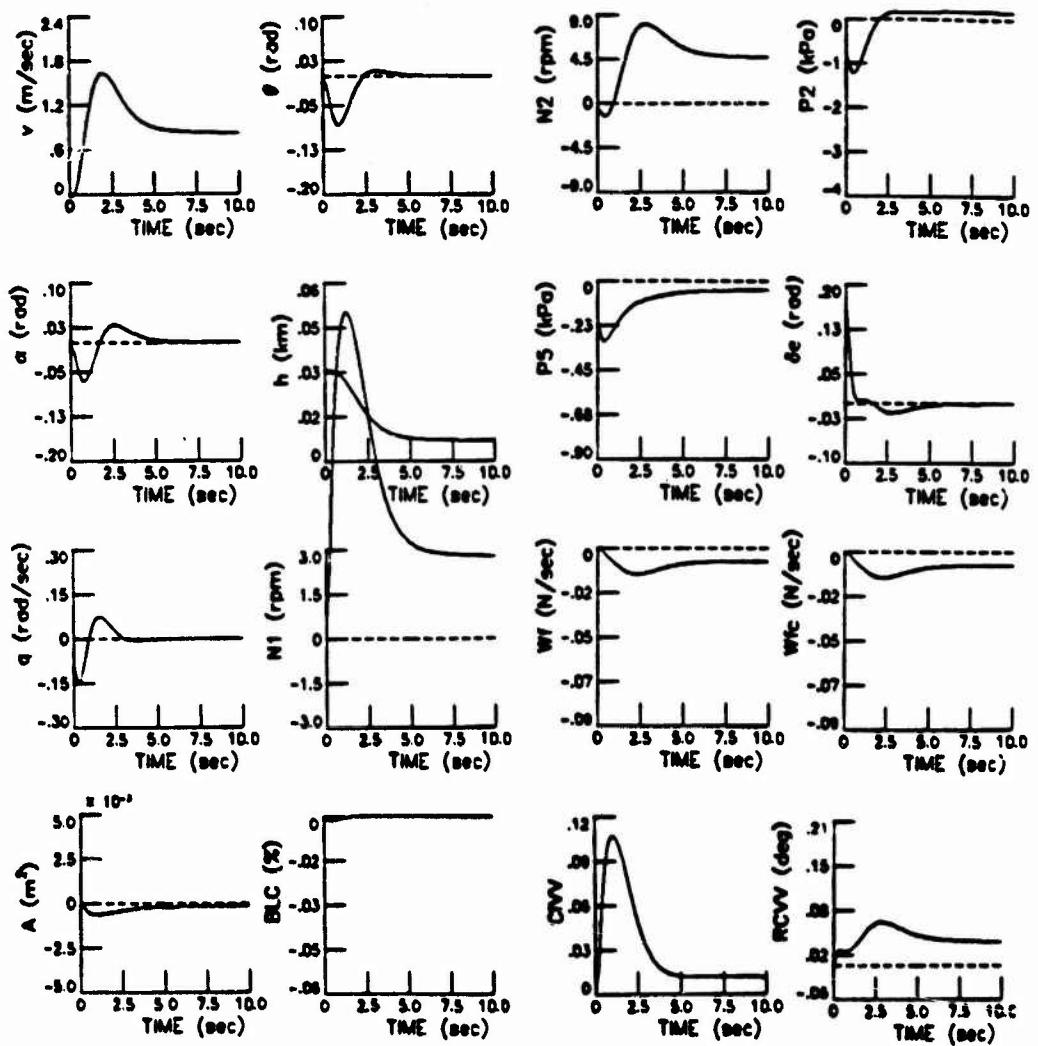


FIG. 4. SUBOPTIMAL DISCRETE-TIME DECENTRALIZED CONTROLLER PERFORMANCE

**-262.470**  
**- 10.07 +j 1.98**  
**- .00263**  
**- 3.37**  
**- 1.86 +j 1.03**  
**- .77**  
**- 1.46 +j 1.12**

TABLE 1. CLOSED-LOOP EIGNEVALUES FOR SYSTEM USING FULL STATE FEEDBACK.  
RAD/SEC

$K_a$	-4.79 E-2	-1.18	5.50 E-1	2.02	4.74
$K_e$	-2.36 E-3	-3.45 E-3	8.52 E-2	-2.30 E-1	-5.95 E-3
	-7.01 E-5	1.52 E-5	1.86 E-3	1.15 E-3	2.52 E-4
	1.13 E-2	-7.78 E-3	-2.35 E-1	4.94 E-1	-1.80 E-2
	3.52 E-3	1.18 E-2	-1.40 E-1	-3.30 E-1	8.05 E-3
	5.36 E-5	2.00 E-4	-1.11 E-3	3.10 E-2	3.52 E-4

TABLE 2. OPTIMAL DECENTRALIZED CONTROL GAINS RESULTING FROM ALGORITHM 1

-20.07  
-10.09 +j 31.42  
- 5.42 +j 1.13  
- 0.0048  
- 1.95 +j 1.60  
- 3.414  
- 1.07 +j .5238

TABLE 3. CLOSED-LOOP EIGENVALUES FOR SYSTEM USING THE OPTIMAL DECENTRALIZED GAINS. RAD/SEC

	<u>SUBSYSTEM ITERATION ALGORITHM</u>	<u>DECENTRALIZED CONTROL ALGORITHM</u>	
	<u>1st Subsystem</u>	<u>2nd Subsystem</u>	
Iterations:	7	37	
	8	37	
	4	37	N/A
	3	14	
	2	2	
	—	—	—
Total Iterations:	24	127	51
Final Cost:	58.253		59.238
Stopping Criterion		1.E-5	1.E-5
Final Alpha:	0.8	0.2	0.3

TABLE 4. CONVERGENCE RATE COMPARISON FOR CONSTRAINED ALGORITHMS

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